(Pages : 6)

P – 1257

Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, September 2022

First Degree Programme under CBCSS

Mathematics

Complementary Course for Physics

MM 1231.1 : MATHEMATICS II – CALCULUS WITH APPLICATIONS IN PHYSICS — II

(2020 Admission)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All **ten** questions are compulsory. Each question carries **1** mark.

1. State De-Moivre's theorem.

2. Find $(3-2i)(-4+3i)$.

3. Define $\cosh x$.

4. What is the condition for $A(x, y)dx + B(x, y)dy = 0$ to be exact?

5. What is the necessary condition to have a stationary point for functions of 2 variables?

6. Write down the Taylor series expansion for functions of 2 variables.

7. Evaluate the integral $\int_1^2 \int_0^4 2xy\, dy\, dx$.

P.T.O.

8. Define Jacobian of x and y with respect to u.

9. Find the velocity of a particle whose motion in space is given by the position vector $r(t) = 2\cos t\ \bar{i} + 2\sin t\ \bar{j} + 5\cos^2 t\ \bar{k}$.

10. Define gradient of a scalar field.

(10 × 1 = 10 Marks)

SECTION – II

Answer **any eight** questions. Each question carries **2** marks.

11. Find the modulus and argument of the complex number $z = 1 + i$.

12. Find the complex conjugate of the complex number $z = w^{2y+2ix}$, where $w = x + 5i$.

13. Prove that $\sin 4\theta = 4\cos^3\theta \sin\theta - 4\cos\theta \sin^3\theta$.

14. Prove that $\cosh^2 x - \sinh^2 x = 1$.

15. Show that $xdy + 3ydx$ is inexact.

16. Define partial derivative of $f(x, y)$ with respect to x.

17. Find the total differential of the function $f(x, y) = x^3 - \dfrac{3y^2}{x}$.

18. Find $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$ if $f(x, y) = x^2 + y^2$.

19. Reverse the order of integration $\int_0^1 \int_0^{1-x} x^2 y\, dy\, dx$.

20. A tetrahedron is bounded by the three co-ordinate surfaces and the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ and has density $\rho(x, y, z)=\rho_0\left(1+\frac{x}{a}\right)$. Find the average value of the density.

21. Evaluate $\int_0^2\int_0^2\int_0^2 xyz\, dx\, dy\, dz$.

22. The position vector of a particle in plane polar co-ordinates is $r(t)=\rho(t)\hat{e}_\rho$. Find an expression for the velocity and acceleration of the particle in these co-ordinates.

23. Find the gradient of $\phi = xy^2z^3$.

24. Find the curl of the vector field $a = x^2y^2z^2\bar{i} + y^2z^2\bar{j} + x^2z^2\bar{k}$.

25. Find the divergence of the vector field $a = x^2y\bar{i} + 2y\bar{j} + 3z\bar{k}$.

26. For the function $\phi = x^2y + yz$ at the point $(1, 2, -1)$, find its rate of change with distance in the direction $a = \bar{i} + 2\bar{j} + 3\bar{k}$.

(8 × 2 = 16 Marks)

SECTION – III

Answer **any six** questions. Each question carries **4** marks.

27. Prove that $\cos 6\theta = 32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1$.

28. Prove that $\sinh^{-1} x = \log\left[x + \sqrt{x^2+1}\right]$.

29. Evaluate $I = \int e^{ax}\cos bx\, dx$.

30. Find the total derivative of $f(x, y) = x^3 + 3xy$ with respect to x, given that $y = \sin^{-1} x$.

31. The temperature of a point (x, y) on a unit circle is given by $T(x, y) = 1 + xy$. Find the temperature of the 2 hottest points on the circle.

32. Find all the second order partial derivatives of the function $f(x, y) = \log(x + y)$.

33. Evaluate $\iint_R (3 - x - y)\,dy\,dx$, where R is the triangular region bounded by x-axis and the lines $y = x$ and $x = 1$.

34. Find the area of the region bounded by $y = x$ and $y = x^2$ in the first quadrant.

35. Find the volume of the tetrahedron bounded by 3 co-ordinate surfaces $x = 0$, $y = 0$, $z = 0$ and the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.

36. Show that $\nabla \cdot (\nabla\phi \times \nabla\psi) = 0$, where ϕ and ψ are scalar fields.

37. Express the vector field $a = yz\bar{i} - y\bar{j} + xz^2k$ in cylindrical polar co-ordinates and hence calculate its divergence.

38. Evaluate $I = \iint_R \left(a + \sqrt{x^2 + y^2}\right) dx\,dy$ where R is the region bounded by the circle $x^2 + y^2 = a^2$.

(6 × 4 = 24 Marks)

SECTION – IV

Answer **any two** questions. Each question carries **15** marks.

39. (a) Prove that $\sin^7\theta = \frac{-1}{2^6}[\sin 7\theta - 7\sin 5\theta + 21\sin 3\theta - 35\sin\theta]$.

(b) Simplify $z = i^{-2i}$.

40. (a) Prove that $\tanh^{-1} x = \frac{1}{2}\log\left(\frac{1+x}{1-x}\right)$.

(b) Show that the function $f(x, y) = x^3 e^{-x^2-y^2}$ has a maximum at the point $\left(\frac{\sqrt{3}}{2}, 0\right)$ and a minimum at $\left(\frac{-\sqrt{3}}{2}, 0\right)$ and a stationary point at the origin.

41. A system contains a very large number N of particles, each of which can be in any of R energy levels with a corresponding energy $E_i, i = 1, 2, \ldots R$. Find the number of particles in the particles amongst the energy levels that maximise the expression $P = \frac{N!}{n_1! n_2! \ldots n_R!}$ subject to the constraints that both the number of particles and total energy remains constant ie, $g = N - \sum_{i=1}^{R} n_i = 0$ and $h = E - \sum_{i=1}^{R} n_i E_i = 0$.

42. Find an expression for a volume element in spherical polar co-ordinates and hence calculate the moment of inertia about a diameter of a uniform sphere of radius 'a' and mass 'M'.

43. (a) Show that the acceleration of a particle travelling along a trajectory $r(t)$ is given by $a(t) = \frac{dv}{dt}\hat{t} + \frac{v^2}{\rho}\hat{n}$, where v is the speed of the particle, $\hat{t}$ is the unit tangent to the trajectory, $\hat{n}$ is the its principal normal and ρ is its radius of curvature.

(b) Find the moment of inertia of a uniform rectangular lamina of mass M with sides a and b, about one of the sides of length 'b'.

44. (a) Find the element of area on the surface of a sphere of radius 'a' and hence calculate the total surface area of the sphere.

(b) At time $t = 0$, the vectors E and B are given by $E = E_0$ and $B = B_0$, where the fixed unit vectors E_0 and B_0 are orthogonal The equations of motion are $\frac{dE}{dt} = E_0 + B \times E_0$, $\frac{dB}{dt} = B_0 + E \times B_0$. Find E and B at a general time t, showing that after a long time, the directions of E and B have almost interchanged.

(2 × 15 = 30 Marks)
