Bishop Moore College

Mavelikara

Project Report

ON

Mathematical Modeling of RC circuit using Computational Techniques

Dissertation submitted to the University of Kerala

in partial fulfilment of the requirement for the award of the Degree of

Bachelor of Science in Physics

By

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CERTIFICATE

This is to certify that the dissertation entitled **"Mathematical Modeling of RC circuit using Computational Techniques"** by **Aysha N.S.** (Reg. No. 23019101008), **Abhishek Nair** (Reg. No: 23019101018), **Vishnu priya T** (Reg. No. 23019101015) and **Malavika.S** (Reg. No. 23019101030), for the award of the degree of Bachelor of Science in Physics is an authentic work under my supervision and guidance during the period from 2019 – 2022.

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2.

Declaration of Originality

We, **Aysha N.S. (Reg. No. 23019101008), Abhishek Nair (Reg. No. 23019101018), Vishnu priya T (Reg. No. 23019101015)** and **Malavika.S (Reg. No. 23019101030)** hereby declare that this dissertation entitled **" Mathematical Modeling of RC circuit using Computational Techniques "** represents our original work carried out as a Bachelor of Science students of University of Kerala and to the best of our knowledge, it contains no material previously published or written by another person, nor any material presented for the award of other degree or diploma of University of Kerala or any other institution. Any contribution made to this research by others, with whom we have worked at University of Kerala or elsewhere, is explicitly acknowledged in the dissertation. Work of other authors cited in this dissertation have been duly acknowledged under the section "Reference". We are fully aware that in case of non-compliance detected in future, the Senate of University of Kerala may withdraw the degree awarded to me on the basis of the present dissertation.

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Chapter 1

INTRODUCTION

Doing experiments is a great way to gain knowledge and increase your critical thinking skills. An example of a good physics experiment would be the charging and discharging of a capacitor through a resistor. As we know, a capacitor can hold a certain amount of charge within itself and can release the charge when it is disconnected from the charging mechanism that was used to charge it. A resistor provides resistance to current flowing through it. A circuit in which a resistor is connected in series with a capacitor is called an RC circuit. In a RC circuit, the capacitor gets charged without any disturbance, when the capacitor is discharged the flow of charge is met with a resistance provided by the resistor, hence the capacitor discharges. Every RC circuit exhibits slightly different results for different values of resistance and capacitance used in the circuit, but every RC circuit follows the same basic principle.

A RC Circuit can be modeled by the use of Runge-Kutta Ordinary Differential Equations (ODEs). This numerical method can be used to obtain the approximate solution of the RC Circuit. Some numerical methods can be used to show ODE systems such as; Euler's method and Runge-Kutta’s method of different orders.

Euler's method is the most basic explicit method for numerical integration of ordinary differential equations and is the simplest Runge-Kutta method. The Euler method is named after Leonhard Euler. Euler's method is a first order method. It is a straight-forward method that estimates the next point based on the rate of change at the current point and it is easy to code. It is a single step method. Notably, Forward Euler's method is unconditionally unstable for un-damped oscillating systems (such as a spring-mass system or wave equations) in space discretization. For complex problems and or boundary conditions it may fail. It can be used for basic numerical analysis. This method is not commonly used for spatial discretization but sometimes used in time discretization. This scheme is not recommended for hyperbolic differential equations because this is more diffusive. Order of convergence of this scheme with grid refinement is very poor.

The forward Euler method is actually the simplest RK method (1 stage, first order). Higher order accurate RK methods are multi-stage because they involve slope calculations at multiple steps at or between the current and next discrete time values. The next value of the dependent variable is calculated by taking a weighted average of these multiple stages based on a Taylor series approximation of the solution. The weights in this weighted average are derived by solving nonlinear algebraic equations which are formed by requiring cancellation of error terms in the Taylor series. Developing higher order RK methods is tedious and difficult without using symbolic tools for computation.

The most popular RK method is RK4 since it offers a good balance between order of accuracy and cost of computation. RK4 is the highest order explicit Runge-Kutta method that requires the same number of steps as the order of accuracy, that is RK1 has one stage, RK2 has 2 stages, RK3 has 3 stages, RK4 has 4 stages, RK5 has 5 stages, and so on. Beyond fourth order, the RK methods become relatively more expensive to compute.

Hence in this research, we will be creating a numerical simulation of the RC Circuit using Runge-Kutta 4th Order (RK4) method compared with the exact solution. We will validate the RC Circuit experimentally using a virtual lab, investigate the charging and discharging process of RC circuit and find out the corresponding values and plot necessary graphs. The purpose of this research is to understand the working of the RC Circuit and the usefulness of a virtual lab.

The report is organized as follows. In Chapter 2, a basic theory of RC circuit, the theory and derivations behind RK4 method is stated. In Chapter 3, a numerical simulation using the Runge-Kutta 4th order method created using octave software is run with the set values and experimental values and a graph is plotted using the program. The set values are put into a setup of RC Circuit in a virtual lab, the results and graphs of both sections are compared and contrasted against.

Finally, in Chapter 4, the future aspects of mathematical modelling and numerical simulation are discussed.

Chapter 2

Theory Section

Theory of the RC circuit

A circuit containing a capacitance [C] and resistance [R] is called a RC circuit. Let a capacitor 'C', a resistance 'R' be connected in series to a D.C source of steady potential and a key 'K'. When the key is in the 'ON' position, a potential is applied throughout the circuit and a charging current 'I' flows through the circuit, the capacitor in the RC circuit undergoes two processes they are "Charging" and "Discharging", will describe below:

Charging

In this case, assume that the capacitor initially does not have any charge. When the potential is applied over the RC circuit system, the charge will flow into the resistor then fill the capacitor. The potential difference across the capacitor increases slowly, till the capacitor is fully charged and charge stops flowing from the battery towards the capacitor. Using Kirchhoff's law we get;

$$dq/dt = 1/R \left[v_0 - (q/C)\right], \qquad (1)$$

where 'dq/dt' is the time rate of change of the charge, 'v_0' is the initial voltage, 'C' is the capacitance, 'R' is the resistance, and 'q' is the charge. To determine the time rate of change of the voltage on the capacitor we can substitute q = Cv in equation (1), equation (1) can be written as;

$$dv/dt = 1/RC\,(v_0 - v), \qquad (2)$$

where 'dv/dt' is the time rate of change of voltage, 'v_0' is the initial voltage and 'v' is the voltage.

Integrating equation (2), we find the solution as;

$$v(t) = v_0 \left(1 - e^{(-t/RC)} \right), \qquad (3)$$

where 'v(t)' is the voltage at a time 't'.

The process of charging the capacitor is shown in the given figures below.

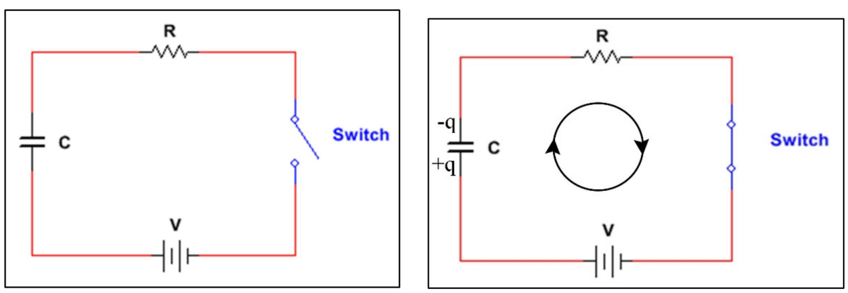


Figure 1 : Charging of the capacitor

Discharging

In this case, the capacitor has a charge 'q' such that $q = C \times v_0$, while the potential on the resistor is "zero". When the capacitor is allowed to discharge, the flow of charge goes through 'R' and hence the process of discharging is only complete after some time. Using Kirchhoff's law,

$$q/C + R\left(dq/dt\right) = 0, \quad (4)$$

where 'q' is the charge, 'C' is the capacitance, 'R' is the resistance, and 'dq/dt' is the time rate of change of the charge.

In the discharging case, to determine the time rate of change of the voltage on the capacitor we can substitute $q = Cv$ from equation (4) can be written as,

$$dv/dt = -v/RC, \qquad (5)$$

where 'dv/dt' is the time rate of change of the voltage and 'v' is the voltage.

Integrating equation (5), the solution is found out as;

$$v(t) = v_0 e^{(-t/RC)}, \qquad (6)$$

where 'v(t)' is the time rate of change of the voltage, 'v_0' is the initial voltage and 't' is the time of discharging. Discharging of the capacitor process shown in the figures below.

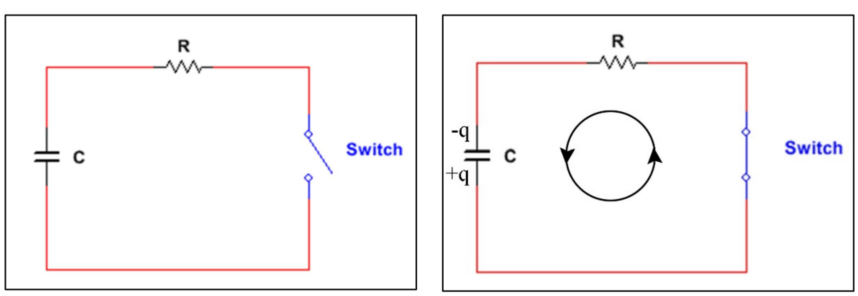


Figure 2 : Discharging of the capacitor

Theory of RK4 method

In this study, we use Runge-Kutta 4 Order Method (RK4) to simulate the charging and discharging of the RC circuit system, as the Runge-Kutta method of solving differential equations allows minimal errors in the solution.

The most widely known member of the Runge–Kutta family is generally referred to as "RK4", the "classic Runge–Kutta method" or simply as "the Runge–Kutta method".

Let an initial value problem be specified as follows:

$$\frac{dy}{dt} = f(t, y), \quad y(t_0) = y_0.$$

Here 'y' is an unknown function that can either be scalar or vector in nature, of time 't', which we would like to approximate; we are told that 'dy/dt', the rate at which 'y' changes, is a function of 't' and of 'y' itself. At the initial time 't_0' the corresponding 'y' value is 'y_0'. The function 'f' and the initial conditions 't_0', 'y_0' are given.

In general a Runge–Kutta method of order 's' can be written as:

$$y_{t+h} = y_t + h \cdot \sum_{i=1}^{s} a_i k_i + \mathcal{O}(h^{s+1})$$

where:

$$k_i = y_t + h \cdot \sum_{j=1}^{s} \beta_{ij} f\left(k_j,\ t_n + \alpha_i h\right)$$

are increments obtained evaluating the derivatives of 'y_t' at the i-th order.

We develop the derivation for the Runge–Kutta fourth-order method using the general formula with 's=4' evaluated, as explained above, at the starting point, the midpoint and the end point of any interval (t, t+h); thus, we choose:

α_i	β_{ij}
$\alpha_1 = 0$	$\beta_{21} = ½$
$\alpha_2 = ½$	$\beta_{32} = ½$
$\alpha_3 = ½$	$\beta_{43} = ½$
$\alpha_4 = 4$	

and '$\beta_{ij} = 0$' otherwise. We begin by defining the following quantities:

$$
\begin{aligned}
y^1_{t+h} &= y_t + hf\left(y_t,\ t\right) \\
y^2_{t+h} &= y_t + hf\left(y^1_{t+h/2},\ t + \frac{h}{2}\right) \\
y^3_{t+h} &= y_t + hf\left(y^2_{t+h/2},\ t + \frac{h}{2}\right)
\end{aligned}
$$

where $y^1_{t+h/2} = \dfrac{y_t + y^1_{t+h}}{2}$ and $y^2_{t+h/2} = \dfrac{y_t + y^2_{t+h}}{2}$.

If we define:

$$
\begin{aligned}
k_1 &= f(y_t,\ t) \\
k_2 &= f\left(y^1_{t+h/2},\ t + \frac{h}{2}\right) = f\left(y_t + \frac{h}{2}k_1,\ t + \frac{h}{2}\right) \\
k_3 &= f\left(y^2_{t+h/2},\ t + \frac{h}{2}\right) = f\left(y_t + \frac{h}{2}k_2,\ t + \frac{h}{2}\right) \\
k_4 &= f\left(y^3_{t+h},\ t + h\right) = f\left(y_t + hk_3,\ t + h\right)
\end{aligned}
$$

and for the previous relations we can show that the following equalities hold up to $\mathcal{O}(h^2)$:

$$\begin{aligned} k_2 &= f\left(y^1_{t+h/2},\, t+\frac{h}{2}\right) = f\left(y_t + \frac{h}{2}k_1,\, t+\frac{h}{2}\right) \\ &= f\left(y_t,\, t\right) + \frac{h}{2}\frac{d}{dt}f\left(y_t,\, t\right) \end{aligned}$$

$$\begin{aligned} k_3 &= f\left(y^2_{t+h/2},\, t+\frac{h}{2}\right) = f\left(y_t + \frac{h}{2}f\left(y_t + \frac{h}{2}k_1,\, t+\frac{h}{2}\right),\, t+\frac{h}{2}\right) \\ &= f\left(y_t,\, t\right) + \frac{h}{2}\frac{d}{dt}\left[f\left(y_t,\, t\right) + \frac{h}{2}\frac{d}{dt}f\left(y_t,\, t\right)\right] \end{aligned}$$

$$\begin{aligned} k_4 &= f\left(y^3_{t+h},\, t+h\right) = f\left(y_t + hf\left(y_t + \frac{h}{2}k_2,\, t+\frac{h}{2}\right),\, t+h\right) \\ &= f\left(y_t + hf\left(y_t + \frac{h}{2}f\left(y_t + \frac{h}{2}f\left(y_t,\, t\right),\, t+\frac{h}{2}\right),\, t+\frac{h}{2}\right),\, t+h\right) \\ &= f\left(y_t,\, t\right) + h\frac{d}{dt}\left[f\left(y_t,\, t\right) + \frac{h}{2}\frac{d}{dt}\left[f\left(y_t,\, t\right) + \frac{h}{2}\frac{d}{dt}f\left(y_t,\, t\right)\right]\right] \end{aligned}$$

where:

$$\frac{d}{dt}f(y_t,\, t) = \frac{\partial}{\partial y}f(y_t,\, t)\dot{y}_t + \frac{\partial}{\partial t}f(y_t,\, t) = f_y(y_t,\, t)\dot{y} + f_t(y_t,\, t) := \ddot{y}_t$$

is the total derivative of 'f' with respect to time.

If we now express the general formula using what we just derived we obtain:

$$\begin{aligned} y_{t+h} = y_t + h \Bigg\{ & a \cdot f(y_t,\, t) + b \cdot \left[f(y_t,\, t) + \frac{h}{2}\frac{d}{dt} f(y_t,\, t) \right] + \\ & + c \cdot \left[f(y_t,\, t) + \frac{h}{2}\frac{d}{dt} \left[f\left(y_t,\, t\right) + \frac{h}{2}\frac{d}{dt} f(y_t,\, t) \right] \right] + \\ & + d \cdot \left[f(y_t,\, t) + h\frac{d}{dt} \left[f(y_t,\, t) + \frac{h}{2}\frac{d}{dt} \left[f(y_t,\, t) + \frac{h}{2}\frac{d}{dt} f(y_t,\, t) \right] \right] \right] \Bigg\} + \mathcal{O}(h^5) \\ = y_t + a \cdot hf_t & + b \cdot hf_t + b \cdot \frac{h^2}{2}\frac{df_t}{dt} + c \cdot hf_t + c \cdot \frac{h^2}{2}\frac{df_t}{dt} + \\ & + c \cdot \frac{h^3}{4}\frac{d^2 f_t}{dt^2} + d \cdot hf_t + d \cdot h^2 \frac{df_t}{dt} + d \cdot \frac{h^3}{2}\frac{d^2 f_t}{dt^2} + d \cdot \frac{h^4}{4}\frac{d^3 f_t}{dt^3} + \mathcal{O}(h^5) \end{aligned}$$

and comparing this with the Taylor series of 'y_{t+h}' around 'y_t' :

$$\begin{aligned} y_{t+h} &= y_t + h\dot{y}_t + \frac{h^2}{2}\ddot{y}_t + \frac{h^3}{6} y_t^{(3)} + \frac{h^4}{24} y_t^{(4)} + \mathcal{O}(h^5) = \\ &= y_t + hf(y_t,\, t) + \frac{h^2}{2}\frac{d}{dt} f(y_t,\, t) + \frac{h^3}{6}\frac{d^2}{dt^2} f(y_t,\, t) + \frac{h^4}{24}\frac{d^3}{dt^3} f(y_t,\, t) \end{aligned}$$

we obtain a system of constraints on the coefficients:

$$\begin{cases} a + b + c + d = 1 \\ \frac{1}{2}b + \frac{1}{2}c + d = \frac{1}{2} \\ \frac{1}{4}c + \frac{1}{2}d = \frac{1}{6} \\ \frac{1}{4}d = \frac{1}{24} \end{cases}$$

which when solved gives

$$a = \frac{1}{6}, b = \frac{1}{3}, c = \frac{1}{3}, d = \frac{1}{6}$$

as stated above.

Now pick a step-size h > 0 and define

$$
\begin{aligned}
y_{n+1} &= y_n + \frac{1}{6} h \left(k_1 + 2k_2 + 2k_3 + k_4\right), \\
t_{n+1} &= t_n + h
\end{aligned}
$$

For n = 0, 1, 2, 3, ..., using

$$
\begin{aligned}
k_1 &= f(t_n, y_n), \\
k_2 &= f\left(t_n + \frac{h}{2}, y_n + h\frac{k_1}{2}\right), \\
k_3 &= f\left(t_n + \frac{h}{2}, y_n + h\frac{k_2}{2}\right), \\
k_4 &= f\left(t_n + h, y_n + hk_3\right).
\end{aligned}
$$

Here 'y_{n+1}' is the RK4 approximation of '$y(t_{n+1})$', and the next value 'y_{n+1}' is determined by the present value 'y_n' plus the weighted average of four increments, where each increment is the product of the size of the interval, 'h', and an estimated slope specified by function 'f' on the right-hand side of the differential equation.

'k_1' is the slope at the beginning of the interval, using 'y' ;

'k_2' is the slope at the midpoint of the interval, using 'y' and 'k_1';

'k_3' is again the slope at the midpoint, but now using 'y' and 'k_2';

'k_4' is the slope at the end of the interval, using 'y' and 'k_3'.

In averaging the four slopes, greater weight is given to the slopes at the midpoint. If 'f' is independent of 'y', so that the differential equation is equivalent to a simple integral, then RK4 is Simpson's rule.

In many practical applications the function 'f' is independent of 't', so the systems of these functions are called autonomous systems, or time-invariant systems, especially in physics, and their increments are not computed at all and are not passed to function 'f', with only the final formula for 't_{n+1}' is used.

Chapter 3

Observations, Discussion and Results

Observations and discussion

Octave programming language is used to create a numerical simulation of RC Circuit. Evaluation of equation (2) gives the RK4 solution function of the charging process,

$$\left.\begin{aligned} v_{i+1} &= v_i + \left(\frac{k_1+2k_2+2k_3+k_4}{6}\right) \\ k_1 &= h\left(\frac{1}{RC}(v_0 - v_i)\right) \\ k_2 &= h\left(\frac{1}{RC}\left(v_0 - \left(v_i + \frac{k_1}{2}\right)\right)\right) \\ k_3 &= h\left(\frac{1}{RC}\left(v_0 - \left(v_i + \frac{k_2}{2}\right)\right)\right) \\ k_4 &= h\left(\frac{1}{RC}\left(v_0 - \left(v_i + \frac{k_3}{2}\right)\right)\right) \end{aligned}\right\} \quad (7)$$

where h = (b-a)/N, 'h' is called step size (N is a positive integer).

Thus, from equation (5), the discharging process has the RK4 solution function,

$$\left.\begin{aligned} v_{i+1} &= v_i + \left(\frac{k_1+2k_2+2k_3+k_4}{6}\right) \\ k_1 &= h\left(\frac{v_i}{RC}\right) \\ k_2 &= h\left(\frac{\left(v_i+\frac{k_1}{2}\right)}{RC}\right) \\ k_3 &= h\left(\frac{\left(v_i+\frac{k_1}{2}\right)}{RC}\right) \\ k_4 &= h\left(\frac{\left(v_i+\frac{k_1}{2}\right)}{RC}\right) \end{aligned}\right\} \quad (8)$$

'v_{i+1}' is used to find the value of voltage at different times. The process will be iterated for N - 1 times. From equation (7) and (8) we set h = 0.1. For R1 = 10 kΩ, R2 = 20 kΩ, C = 10 µF and v0 = 10 volt. Lastly, we will arrive at the approximate value for the solution of the RC circuit system.

After calculating the numerical simulation, the solution will be plotted using octave programming language. Then, we can investigate the graphic form of charging and discharging process of the RC Circuit. The graphs are obtained as follows,

Graph showing charging process

With different values of resistance

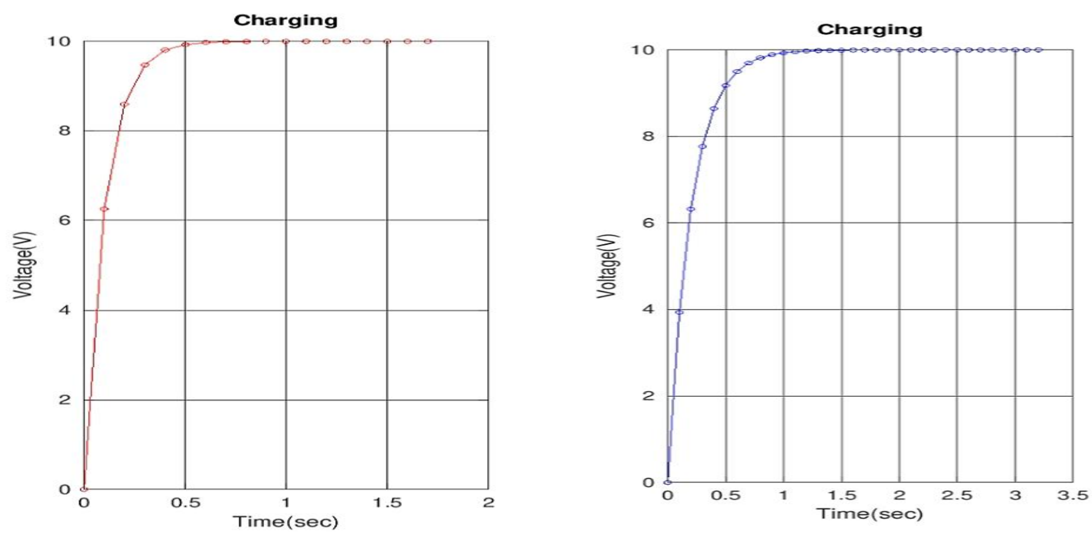


Figure 3 : Charging of RC circuits with different resistors

In this graph, the voltage passing through the capacitor increases rapidly. The red line with red dots represents the resistor capacitor combination, with resistance of the resistor being 10 kΩ. The blue line with blue dots represents the resistor capacitor combination, with resistance of the resistor being 20 kΩ.

Comparing the graphs, we see that the circuit having resistor with higher resistance takes more time to charge.

With different values of capacitance

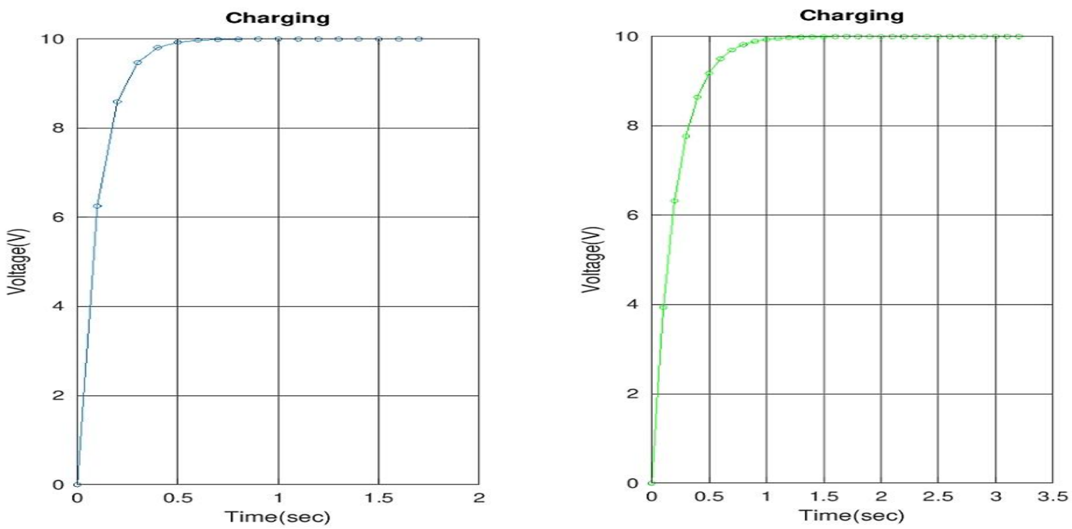


Figure 4 : Charging of RC circuit with different capacitors

Resistance used in both cases is 10kΩ. The capacitance used in the first case represented by light blue lines and light blue dots is 10μF. The capacitance used in the second case represented by green line and green dots is 20μF. Comparing the graphs, we see that in the first case the charging happens much faster than in the second case. This is because the capacitor used in the second case can hold more charge compared to the capacitor used in the first case.

Theoretically, it takes infinite time for the potential to attain its maximum value. But in practice after a time of five times the time constant, the potential reaches its maximum value approximately.

The time constant of the RC circuit is defined as the time taken by the capacitor to acquire 63.2% of its maximum charge.

Graph showing discharging process

With different values of resistance

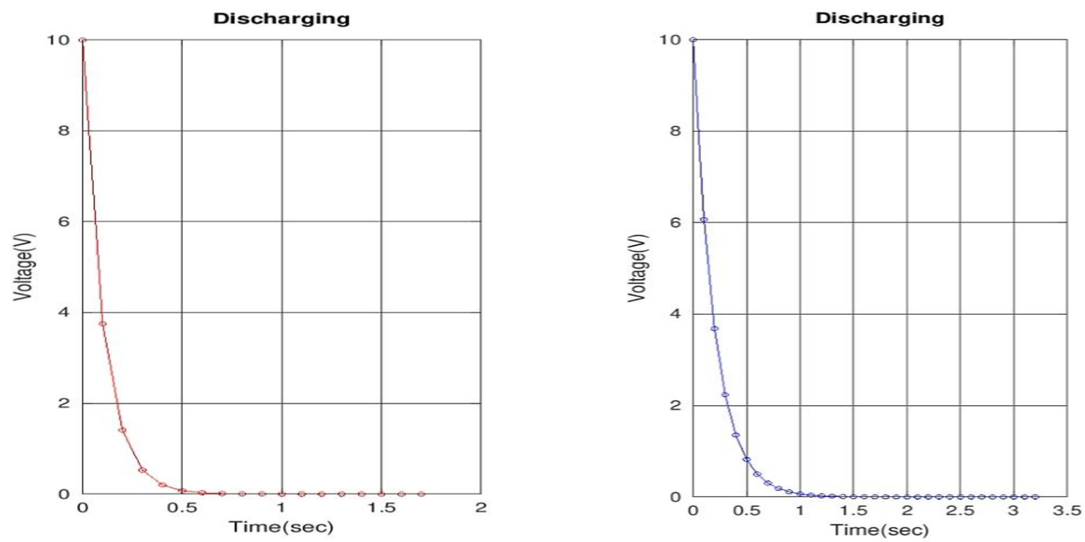


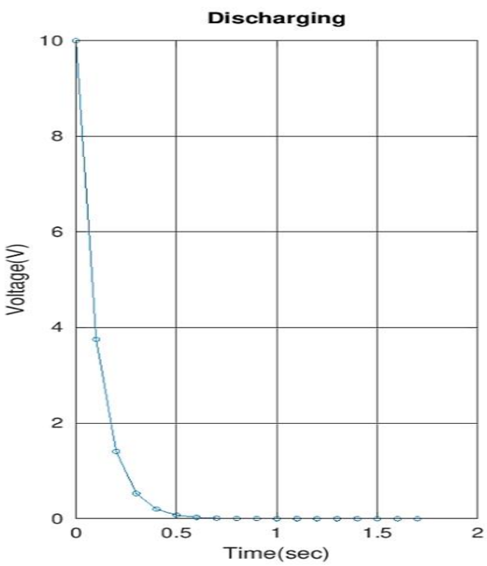
Figure 5 : Discharging of RC circuits with different resistors

In this graph, we can see that the voltage is much lower than when it was charging. The red line with red dots represents the resistor capacitor combination, with resistance of the resistor being 10 kΩ. The blue line with blue dots represent the resistor capacitor combination, with resistance of the resistor being 20 kΩ.

In between the charging and discharging, the graphs falls at a very steep pace, this means that the decreasing potential across the capacitor is exponential.

Comparing the graphs, we see that the circuit having resistor with higher resistance takes more time to discharge.

With different values of capacitance



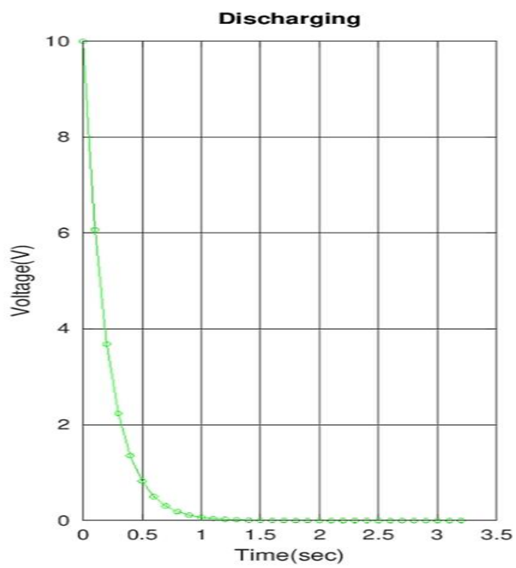


Figure 6 : Discharging of RC circuit with different capacitors

Resistance used in both cases is 10kΩ. The capacitance used in the first case represented by light blue lines and light blue dots is 10μF. The capacitance used in the second case represented by green line and green dots is 20μF. Comparing the graphs, we see that in the first case it takes less time to discharge than in the second case. The reason behind this is that the capacitor used in the second case holds more charge and hence takes more time to completely remove the charge stored inside it.

The time constant of an RC circuit can also be defined as the time taken by the charge on the capacitor to decrease from its maximum value to 0.368 times its maximum.

Similar to the decay of potential across the capacitor, the decay of charge with time is also exponential.

On the Virtual lab

We repeat the experiment with the same values, that is , v0 = 10 V, C = 10 μF for resistances of 10 kΩ and 20 kΩ, also for the system of circuits with the same resistance of 10kΩ

Charging and Discharging of RC circuit with resistor of resistance 10 kΩ

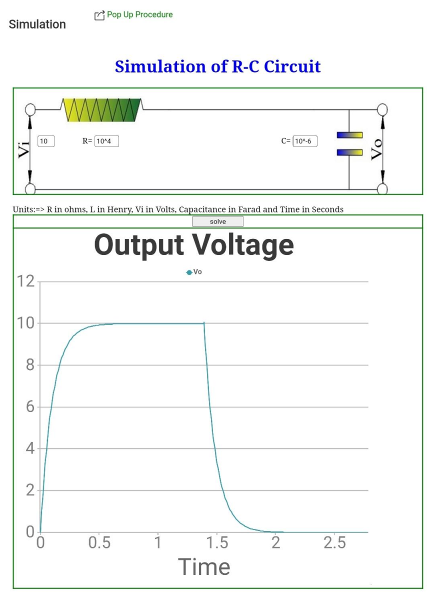
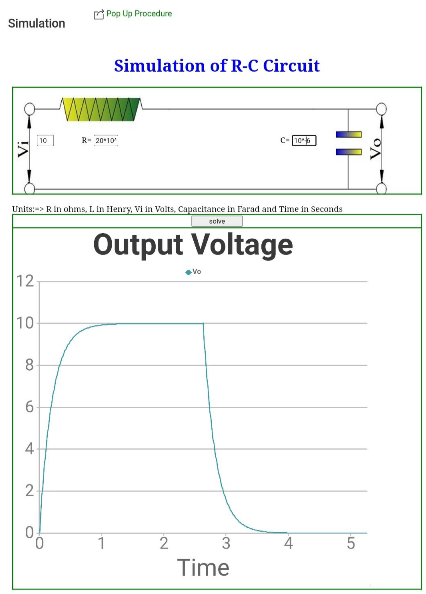


Figure 7 : Charging and discharging of RC circuit with resistance of 10kΩ

Here, 'Vi' is the input voltage which means that 'Vi' takes the same value as 'v_0' and 'Vo' is the output voltage which means that 'Vo' takes the same value as 'v'.

The increase and decrease of potential across the capacitor connect in series with the resistor of resistance 10 kΩ is exponential.

<u>Charging and discharging of RC circuit with resistor of resistance 20 kΩ</u>



<u>Figure 8</u> : Charging and discharging of RC circuit system with resistance of 20kΩ

Here, 'Vi' is the input voltage which means that 'Vi' takes the same value as 'v_0' and 'Vo' is the output voltage which means that 'Vo' takes the same value as 'v'.

The increase and decrease of the potential across the capacitor connected in series with a resistor of resistance 20 kΩ is exponential.

Comparing the two graphs, we see that the system of circuits using a resistor with higher resistance takes more time to charge and discharge.

Charging and discharging of RC circuit with capacitor of capacitance 10μF

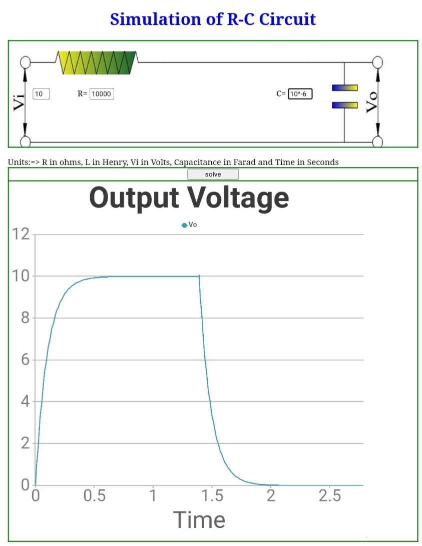


Figure 9 : Charging and discharging of RC circuit with capacitance of 10μF

Here, 'Vi' is the input voltage which means that 'Vi' takes the same value as 'v_0' and 'Vo' is the output voltage which means that 'Vo' takes the same value as 'v'.

The resistance used is 10kΩ.

Charging and discharging of RC circuit with capacitor of capacitance 20μF

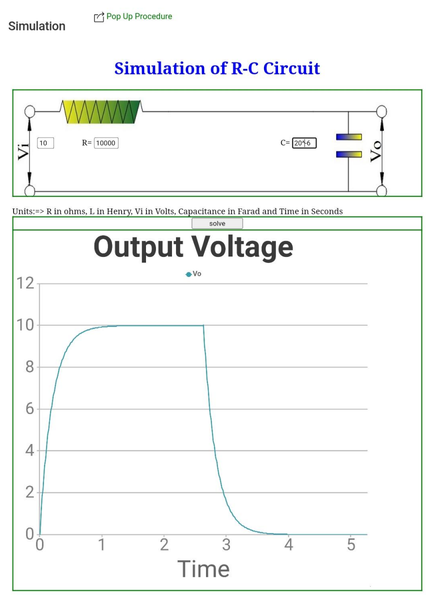


Figure 10 : Charging and discharging of RC circuit with capacitance of 20μF

Here, 'Vi' is the input voltage which means that 'Vi' takes the same value as 'v_0' and 'Vo' is the output voltage which means that 'Vo' takes the same value as 'v'.

The resistance used is 10kΩ.

Comparing both graphs, we see that the system of circuit using a capacitor with higher capacitance takes more time to charge and discharge.

Results

In this research, we have studied the RC circuit system. This research shows that the RK4 method can be used to successfully solve the RC circuit system, showing the best agreement when compared to the theoretical solution.

Using Octave programming language, a numerical simulation of RK4th order method is created and the values corresponding to the concerning RC circuit systems are input into the simulation and using a virtual lab, those same values are used and the results are compared, thus we have verified experimentally, with an excellent quantitative agreement, the mathematical model and the numerical simulation that describes the charging or discharging of the capacitor in series with a resistor.

Comparing the graphs plotted using Octave programming language and graphs plotted using a virtual lab, we see that the graphs plotted using Octave are similar to the graphs plotted using the virtual lab. Hence, we can confirm that a virtual lab can provide the same experience as an ordinary lab.

Chapter 4

Future Aspects of Mathematical modelling

Models describe our beliefs about how the world functions. A mathematical model can be defined as an abstract model which uses mathematical language to describe the behavior and evolution of a system. Mathematical models are used widely in the many different sciences and engineering disciplines such as physics, biology, chemistry and engineering. Mathematical models may have many different forms, including continuous time and discrete time dynamical systems using differential equations and difference equations respectively, statistical models, partial differential equations, or game theoretic models. Mathematical modeling has an important role in discovering the problems which occur in our daily life. Mathematical and computational models have been frequently used to help interpret experimental data. Models also can help to describe our beliefs about how different phenomena around the world function. In mathematical modeling, we try to transfer those beliefs and pictures into the language of mathematics. This transformation is very beneficial. First, Mathematics is an exact and delicate language. Second, we can easily formulate ideas and also determine the basic assumptions. The governed rules in Mathematics help us to manipulate the problem. Strongly speaking, in Mathematical modeling, we are using the results which have been already proved by mathematicians over hundreds of years. Computers play an important role in performing numerical simulations and calculations. Mathematics modelling has many advantages. Some of which are:

1. Since mathematics is a very precise language. This helps us to formulate ideas and identify underlying assumptions.
2. Mathematics is a concise language, with well-defined rules for manipulations.
3. All the results that mathematicians have proved over hundreds of years are at our disposal.
4. Computers can be used to perform numerical calculations.

The majority of interacting systems in the real world are far too complicated to model in their entirety, this is a large element of compromise in mathematical modelling. We can solve this problem by identifying the most important parts of the system and then we include them in the model, the rest will be excluded. Hence the first level compromise is to identify the most important parts of the system. Afterward, computer simulations can be applied to handle the model equations and desired manipulations. The second level of compromise concerns the amount of mathematical manipulation which is worthwhile. Although mathematics has the potential to prove general results, these results depend critically on the form of equations used. Small changes in the structure of equations may require enormous changes in the mathematical methods. Using computers to handle the model equations may never lead to elegant results, but it is much more robust against alterations.

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