

Reg No.:

Name :



U8345



# University of Kerala

First Semester Degree Examination, November 2024

Four Year Undergraduate Programme

Discipline Specific Course

## Mathematics

UK1DSCMAT100, Foundations of Mathematics

Academic Level: 100-199

Time: 2 hours

Max. Marks: 56

Part A. 6 Marks. Time: 5 Minutes

Objective Type. 1 Mark Each. Answer all Questions

(Cognitive Level: Remember/Understand)

Qn. No.	Question	Cognitive Level	Course Outcome (CO)
1.	Define an anti-symmetric relation.	Remember	CO4
2.	A matrix $A$ is said to be non singular if..	Remember	CO1
3.	The determinant of the matrix $A = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$ is	Understand	CO1
4.	A homogeneous linear system of $n$ equations with $n$ unknowns has a unique solution if...	Understand	CO2
5.	The linear congruence $ax \equiv b \pmod{m}$ has a unique solution if and only if	Remember	CO3
6.	The sum, $\sum_{i=1}^n (2i - 1)$ is	Remember	CO3

**Part B. 10 Marks. Time:20 Minutes**  
Two-Three sentences. 2 Marks Each. Answer all Questions  
(Cognitive Level: Remember/Understand/Apply)

Qn. No.	Question	Cognitive Level	Course Outcome (CO)
7.	Define one-to-one function. Give an example.	Remember	CO4
8.	Show that for any square matrix $A$ , $\frac{1}{2}(A + A^t)$ is always symmetric, where $A^t$ is the transpose of $A$ .	Remember	CO1
9.	Express (28, 12) as a linear combination of 28 and 12.	Remember	CO3
10.	Find $gcd$ of 120 and 28.	Understand	CO3
11.	State Rouché's theorem. Give an example of a system of equations which is inconsistent.	Apply	CO2

**Part C. 16 Marks. Time:35 Minutes**  
Short-Answer. 4 Marks Each. Answer all Questions, choosing among options within each question.  
(Cognitive Level: Understand/Analyse/Apply)

Qn. No.	Question	Cognitive Level	Course Outcome (CO)
12.	<p>A.) Show that the relation <math>\equiv</math> is an equivalence relation in the set of all integers.</p> <p align="center"><b>OR</b></p> <p>B.) Define congruence relation. The equivalence relation <math>\equiv</math> on the set of integers defined by <math>xRy</math> if <math>x \equiv y \pmod{4}</math>. Find all equivalence classes under this relation.</p>	Understand	CO4
13.	<p>A.) If <math>\begin{vmatrix} a &amp; a^2 &amp; a^3 - 1 \\ b &amp; b^2 &amp; b^3 - 1 \\ c &amp; c^2 &amp; c^3 - 1 \end{vmatrix} = 0</math>, in which <math>a, b, c</math> are different, show that <math>abc = 1</math>.</p> <p align="center"><b>OR</b></p> <p>B.) Express <math>\begin{bmatrix} 3 &amp; 5 &amp; -7 \\ -8 &amp; 11 &amp; 4 \\ 13 &amp; -14 &amp; 6 \end{bmatrix}</math> as the sum of a lower triangular matrix with zero leading diagonal and an upper triangular matrix.</p>	Apply	CO1

Qn. No.	Question	Cognitive Level	Course Outcome (CO)
14.	<p>A.) For what value of <math>\lambda</math>, the system of equation</p> $2x + 3y + 5z = 9, 7x + 3y - 2z = 8, 2x + 3y + \lambda z = 1$ <p>has unique solution?</p> <p style="text-align: center;">OR</p> <p>B.) Find the values of <math>k</math> for which the system of equations</p> $\begin{aligned}(3k - 8)x + 3y + 3z &= 0 \\ 3x + (3k - 8)y + 3z &= 0 \\ 3x + 3y + (3k - 8)z &= 0.\end{aligned}$ <p>has a non-trivial solution.</p>	Analyse	CO2
15.	<p>A.) Find the remainder when <math>3^{181}</math> is divided by 17.</p> <p style="text-align: center;">OR</p> <p>B.) Using canonical decomposition of 1050 and 2574, find their lcm.</p>	Understand	CO3

**Part D. 24 Marks. Time:60 Minutes**

Long-Answer. 6 Marks Each. Answer all 4 Questions, choosing among options within each question.  
(Cognitive Level: Understand/Analyse/ Apply)

Qn. No.	Question	Cognitive Level	Course Outcome (CO)
16.	<p>A) Find the number of positive integers in the range 1976 through 3776 that are; (i.) Divisible by 13 or 15. (ii.) Not divisible by 15 or 17.</p> <p style="text-align: center;">OR</p> <p>B) Using Euclidean algorithm find <math>(4076, 1024)</math> and express <math>(4076, 1024)</math> as a linear combination of 4076 and 1024.</p>	Understand	CO3