U8346

University of Kerala

First Semester Degree Examination, November 2024

Four Year Undergraduate Programme

Discipline Specific Course

Mathematics

UK1DSCMAT100, Foundations of Mathematics

Academic Level: 100-199

Time: 2 hours   Max. Marks: 56

Part A. 6 Marks. Time:5 Minutes
Objective Type. 1 Mark Each. Answer all Questions
(Cognitive Level: Remember/Understand)

Qn. No.	Question	Cognitive Level	Course Outcome (CO)
1.	State De-Morgan's law..	Remember	CO 1
2.	What is the normal form of a matrix?	Remember	CO 1
3.	List the elements of the set $A = \{x \in N : x \text{ is even}, x < 11\}$	Understand	CO 1
4.	Find the rank of the matrix $\begin{bmatrix} 1 & 2 \\ 1 & 4 \end{bmatrix}$.	Understand	CO 2
5.	State Well-Ordering Principle.	Remember	CO 3
6.	Define equivalence classes assosciated with an equivalence relation.	Remember	CO 4

Qn. No.	Question	Cognitive Level	Course Outcome (CO)
7.	Define equivalence relation on a set and give an example.	Remember	CO 4
8.	Write 2 properties of determinants.	Remember	CO 1
9.	Compute the inverse of $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$.	Remember	CO 1
10.	Construct the Cartesian product $A \times B$ for $A = \{1, 2\}$ and $B = \{x, y\}$.	Understand	CO 4
11.	Apply the euclidean algorithm to find $(4076, 1024)$.	Apply	CO 3

Part C. 16 Marks. Time:35 Minutes
Short-Answer. 4 Marks Each. Answer all Questions, choosing among options within each question.
(Cognitive Level: Understand/Analyse/Apply)

Qn. No.	Question	Cognitive Level	Course Outcome (CO)
12.	A) Solve the congruence $5x \equiv 3 \pmod 7$. OR B) Prove that $a \equiv b \pmod m$ if and only if $a = b + km$ for some integer k.	Understand	CO 3
13.	A) Express $\begin{bmatrix} 4 & 2 & -3 \\ 1 & 3 & -6 \\ -5 & 0 & -7 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix. OR B) Find the matrix A if $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix}$.	Understand	CO 1.
14.	A) Find the minors and cofactors of $A = \begin{bmatrix} 3 & 1 & -4 \\ 2 & 5 & 6 \\ 1 & 4 & 8 \end{bmatrix}$. OR B) By mathematical induction, prove that if $A = \begin{bmatrix} 11 & -25 \\ 1 & -9 \end{bmatrix}$, then $A^n = \begin{bmatrix} 1 + 10n & -25n \\ 4n & 1 - 10n \end{bmatrix}$.	Analyze	CO 1

15.	A) Given a function $f : A \rightarrow B$ where $A = \{1, 2, 3\}$ and $B = \{a, b\}$, analyze if it can be a one-to-one function. Justify.	Apply	CO 4
	OR		
	B) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ and $g : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 2x + 1$ and $g(x) = x^2 - 2$. Find $f \circ g$ and $g \circ f$.		

Part D. 24 Marks. Time:60 Minutes

Long-Answer. 6 Marks Each. Answer all 4 Questions, choosing among options within each question.
(Cognitive Level: Understand/Analyse/ Apply)

Qn. No.	Question	Cognitive Level	Course Outcome (CO)
16.	A) State and prove division theorem.	Understand	CO 3
	OR		
	B) Let $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$. Prove that $a + c \equiv b + d \pmod{m}$ and $ac \equiv bd \pmod{m}$.		
17.	A) Test for consistency and solve $5x + 3y + 7z = 4$, $3x + 26y + 2z = 9$, $7x + 2y + 10z = 5$.	Understand	CO 2.
	OR		
	B) Investigate the values of λ and μ so that the equations $2x + 3y + 5z = 9$, $7x + 3y - 2z = 8$, $2x + 3y + \lambda z = \mu$, have (i) no solution(ii)a unique solution(iii)an infinite number of solutions.		
18.	A) Design a function that is both one-to-one and onto. Provide its definition and a graph illustrating the mapping.	Analyse	CO 4
	OR		
	B) Analyze the properties of the relation $R = \{(1, 1), (2, 2), (1, 2), (2, 1)\}$. Is it an equivalence relation? Justify your answer.		
19.	A) Solve the equations $3x + y + 2z = 3$, $2x - 3y - z = -3$, $x + 2y + z = 4$ by (i)determinants(ii)matrices.	Apply	CO 2
	OR		
	B) Reduce the matrix $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ in to its normal form and hence find its rank.		