U8350

University of Kerala

First Semester Degree Examination, November 2024

Four Year Undergraduate Programme

Discipline Specific Course

Mathematics

UK1DSCMAT101, Differential Calculus and Linear Algebra

Academic Level: 100-199

Time: 2 hours

Max. Marks: 56

Part A. 6 Marks. Time:5 Minutes
Objective Type. 1 Mark Each. Answer all Questions
(Cognitive Level: Remember/Understand)

Qn. No.	Question	Cognitive Level	Course Outcome (CO)
1.	What is the average velocity of the particle over a time interval $[t_0, t_0 + h], h > 0$?	Remember	CO2
2.	Define a concave up function on an open interval?	Remember	CO2
3.	What is critical point of a function?	Understand	CO1
4.	Find the rank of the matrix $\begin{bmatrix} 4 & -2 & 2 \\ -2 & 1 & -1 \\ 2 & -1 & 1 \end{bmatrix}$.	Understand	CO3
5.	State the condition for a linear system $AX = B$ of m equations in n unknowns have unique solution.	Remember	CO3
6.	If 2, 5 and 7 are the eigenvalues of a 3×3 matrix A, then what the eigenvalues of A^T.	Remember	CO3

Part B. 10 Marks. Time:20 Minutes
Two-Three sentences. 2 Marks Each. Answer all Questions
(Cognitive Level: Remember/Understand/Apply)

Qn. No.	Question	Cognitive Level	Course Outcome (CO)
7.	Suppose f is concave down on an open interval, what about $-f$? Justify your answer.	Remember	CO1
8.	Prove that $f(x) = x^3$ has a point of inflection at $x = 0$.	Understand	CO2
9.	Prove that the function $f(x) = x^2 - 4x + 3$ is concave up on the interval $(-\infty, \infty)$	Understand	CO2
10.	Do the equations $$3x + 2y = 0, 6x + 4y = 0$$ have a non-trivial solution? Why?	Understand	CO3
11.	Find the eigenvalues of the matrix $A = \begin{vmatrix} 1 & 6 \\ 5 & 2 \end{vmatrix}$.	Understand	CO3

Part C. 16 Marks. Time:35 Minutes
Short-Answer. 4 Marks Each. Answer all Questions, choosing among options within each question.
(Cognitive Level: Understand/Analyse/Apply)

Qn. No.	Question	Cognitive Level	Course Outcome (CO)
12.	A) Prove that $f(x) = |x|$ is not differentiable at $x = 0$. **OR** B) If a function $f(x)$ is differentiable at x= a ,then prove that $f(x)$ is continuous at $x = a$.	Understand	CO1
13.	A) Find the intervals on which $f(x) = x^2 - 6x + 5$ is increasing and the intervals on which it is decreasing. **OR** B) A garden is to be laid out in a rectangular area and protected by a chicken wire fence. What is the largest possible area of the garden if only 100 running feet of chicken wire is available for the fence?	Apply	CO2

Question	Cognitive Level	Course Outcome (CO)
4. A) Solve the system of equations using Gauss elimination method $$x + y + z = 6,$$ $$x + 2y - 3z = -4,$$ $$-x - 4y + 9z = 18$$ **OR** B) Using Cramer's Rule, solve the system of equations $$x + y + z = 6,$$ $$y + 3z = 11,$$ $$x - 2y + z = 0$$	Apply	CO3
15. A) Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ **OR** B) Examine whether the matrix $A = \begin{bmatrix} 1 & -3 & 3 \\ 0 & -5 & 6 \\ 0 & -3 & 4 \end{bmatrix}$ is diagonalizable.	Apply	CO3

Part D. 24 Marks. Time:60 Minutes

Long-Answer. 6 Marks Each. Answer all 4 Questions, choosing among options within each question.
(Cognitive Level: Understand/Analyse/ Apply)

Qn. No.	Question	Cognitive Level	Course Outcome (CO)
16.	A) Find the absolute maximum and minimum values of the function $f(x) = 2x^3 - 15x^2 + 36x$ on the interval $[1, 5]$, and determine where these values occur. **OR** B) Find the radius and height of the right circular cylinder of largest volume that can be inscribed in a right circular cone with radius 6 inch and height 10 inches.	Apply	CO3

17.	A) Prove that $f(x) = \ln(x)$ is differentiable for $x > 0$, using definition of derivative.	Apply	CO2
	OR		
	B) If $4x^2 - 2y^2 = 9$, using implicit differentiation method, find $\dfrac{d^2y}{dx^2}$.		
18.	A) Show that the equations	Apply	CO3
	$$x + 2y + z = 3,$$ $$2x + 3y + 2z = 5,$$ $$3x - 5y + 5z = 2$$ are consistent and solve the same.		
	OR		
	B) Find the values of λ for which the system of equations $$x + y + z = 1,$$ $$x + 2y + 4z = \lambda,$$ $$x + 4y + 10z = \lambda^2$$ will be consistent.		
19.	A) Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$.	Apply	CO3
	OR		
	B) Find the matrix that diagonalize the matrix $A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$.		