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Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, March 2022

First Degree Programme under CBCSS

Mathematics

Complementary Course for Physics

MM 1331.1 : MATHEMATICS III DIFFERENTIAL EQUATIONS, THEORY OF EQUATIONS AND THEORY OF MATRICES

(2014 – 2017 Admissions)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the **first ten** questions are compulsory. They carry **1** mark each.

1. What is a coincident root?

2. If α, β, γ and δ are the roots of the equation $3x^4+2x^3+5x^2+7x-2=0$, what is the value of $\alpha+\beta+\gamma+\delta$?

3. State the Fundamental Theorem of Algebra for a polynomial of degree n?

4. Find the real root of the cubic equation $x^3+22x+52=0$, one root being $1+5i$

5. What is meant by modeling of a differential equation?

6. Solve the initial value problem: $y'=3y$, $y(0)=5.7$.

P.T.O.

7. What is a basis for a vector space?

8. Define row rank of a matrix.

9. Define the characteristic polynomial of a matrix.

10. Check whether the matrix $\begin{bmatrix} 1 & 0 & -2 \\ 2 & 4 & 5 \\ 0 & 0 & 2 \end{bmatrix}$ is singular or not.

(10 × 1 = 10 Marks)

SECTION – II

Answer **any eight** questions. These questions carry **2** marks each.

11. Solve the equation $x^3 - 8x^2 + 9x + 18 = 0$, given that sum of two of its roots is 5.

12. Form a rational cubic equation whose roots are 5 and $1+4i$.

13. Solve the equation $x^3 + 3x^2 - 6x - 8 = 0$, given that the roots are in geometric progression.

14. Express $f(x) = x^3 + 6x^2 - 4x + 5 = 0$ in reduced form.

15. Solve the initial value problem: $yy' + 4x = 0, \ y(0) = 3$.

16. Define an exact differential equation.

17. Solve: $y' - y = e^{2x}$.

18. State the existence theorem for the solution of a first order differential equation.

19. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$, Show that $A^2 - A - 6I = 0$.

20. Find the Eigen values of the matrix $\begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$.

21. Find the rank of the matrix $A=\begin{bmatrix} 2 & 0 & 1 \\ 0 & -4 & 6 \\ 2 & -2 & 5 \end{bmatrix}$ by reducing it to echelon form.

22. Let $A=\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, find the A^2 using the Cayley Hamilton Theorem.

(8 × 2 = 16 Marks)

SECTION – III

Answer **any six** questions. These questions carry **4** marks each.

23. Find a real root of the equation: $x^3+2x+5=0$.

24. Describe the bisection method of finding a root of the equation $f(x)=0$.

25. Solve the equation $x^3-3x-4=0$, using Newton-Raphson method correct to 3 significant figures.

26. Solve : $\cos(x+y)dx+(3y^2+2y+\cos(x+y))\,dy=0$

27. Solve the IVP : $y'=3x^2-\frac{y}{x}$, $y(1)=5$

28. Explain about orthogonal Trajectories.

29. Solve : $x^2y''+axy'+\frac{1}{4}(1-a)^2y=0$.

30. Prove that if an $n \times n$ matrix A has n distinct eigen values, then it has n linearly independent Eigen vectors.

31. State and prove orthogonality of Eigen vectors.

(6 × 4 = 24 Marks)

SECTION – IV

Answer **any two** questions. These questions carry **15** marks each.

32. Solve the equation $e^x + x - 2 = 0$ giving the root to 6 significant figures using Newton-Raphson method.

33. (a) Solve the initial value problem : $y' + y \tan x = \sin 2x,\ y(0) = 1$

    (b) Solve the initial value problem : $y' = 1 + y^2,\ y(0) = 0$ under the rectangle $|x| < 5,\ |y| < 3$.

34. State and prove fundamental theorem for the homogeneous linear differential equations (order).

35. Prove that $A = \begin{bmatrix} -1 & 1 & 3 \\ 2 & 1 & 4 \\ 1 & 0 & -2 \end{bmatrix}$ is diagonalizable and find the diagonal form.

(2 × 15 = 30 Marks)