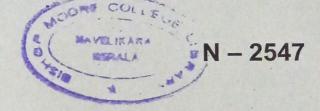
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Name :

Third Semester B.Sc. Degree Examination, March 2022 First Degree Programme under CBCSS

Mathematics

Complementary Course for Physics

MM 1331.1 – MATHEMATICS III – CALCULUS AND LINEAR ALGEBRA (2018 Admission)

Time: 3 Hours Max. Marks: 80

SECTION - I

All the ten questions are compulsory. They carry 1 mark each.

- Define degree of an ODE.
- 2. What is the auxillary equation of $\frac{d^2y}{dx^2} 2\frac{dy}{dx} + y = e^x$.
- 3. Write the general form of Bernoulli's equation.
- 4. Prove that $\vec{a} = (xy^2 + z)\vec{i} + (x^2y + 2)\vec{j}$ is conservative.
- 5. State Stoke's theorem.
- 6. Find the average value of $\sin x$ on $(-\pi, \pi)$.
- 7. What are the Fourier coefficients of an even function f(x) in the interval (-1, 1).

- 9. Find the rank of the matrix $\begin{pmatrix} 1 & 1 & 2 \\ 2 & 4 & 6 \\ 3 & 2 & 5 \end{pmatrix}$.
- 10. Find the trace of the matrix $\begin{pmatrix} 1 & 0 & -1 \\ 2 & -3 & 0 \\ 1 & -3 & 2 \end{pmatrix}$.

SECTION - II

Answer any eight questions from among the questions 11 to 22. These questions carry 2 marks each.

- 11. Solve $x \frac{dy}{dx} + 3x + y = 0$.
- 12. Solve $y = px + p^2$
- 13. Find the complementary function of the equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 4e^{-x}$.
- 14. Evaluate the line integral $I = \int \vec{a} \cdot d\vec{r}$ where $\vec{a} = (x+y)\vec{i} + (y-x)\vec{j}$ along the parabola $y^2 = x$ from (1, 1) to (4, 2) in the xy-plane.
- 15. Find the vector area of the surface of the hemisphere $x^2 + y^2 + z^2 = a^2$, $z \ge 0$, by evaluating the line integral $S = \frac{1}{2} \oint_C \vec{r} \times d\vec{r}$ around its perimeter.
- 16. Define even and odd functions and give examples.
- 17. What are the Dirichlet conditions for the existence of the Fourier series of a periodic function?
- Define Fourier transform.

- 19. Find the direction of the line of intersection of the plane x-2y+3z=4 and 2x+y-z=5.
- 20. Show that the functions 1, x, sin x are linearly independent.
- 21. Find the inverse of the matrix $\begin{pmatrix} 2 & 1 \\ 0 & -3 \end{pmatrix}$.
- 22. Define symmetric matrix and prove that AA^T is a symmetric matrix for any matrix A.

SECTION - III

Answer any six questions from among the following questions 23 to 31. These questions carry 4 marks each.

- 23. Solve: $\frac{dy}{dx} = -\frac{2}{y} \frac{3y}{2x}$.
- 24. Solve: $\frac{dy}{dx} = (x+y+1)^2$.
- 25. Solve $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} 4y = 0$.
- 26. The vector field \vec{F} is given by $\vec{F} = 2xz\vec{i} + 2yz^2\vec{j} + (x^2 + 2y^2z 1)\vec{k}$. Calculate $\nabla \times \vec{F}$ and deduce that \vec{F} can be written $\vec{F} = \nabla \phi$. Determine the form of ϕ .
- 27. Show that the area of a region R enclosed by a closed curve C is given by $A = \frac{1}{2} \oint x \, dy y \, dx = \oint_C x \, dy = -\oint_C y \, dx.$
- 28. Find the Fourier series of the function

$$f(x) = \begin{cases} 0 & -\pi < x < 0 \\ 1 & 0 < x < \pi \end{cases}$$

- 29. Expand $f(x) = \begin{cases} -1 & -l < x < 0 \\ 1 & 0 < x < l \end{cases}$ as a Fourier series.
- 30. Use Crammer's rule to solve 2x z = 2, 6x + 5y + 3z = 7; 2x y = 4.
- 31. Find the eigen values and eigen vector of $\begin{pmatrix} 1 & 2 & 2 \\ 2 & 3 & 0 \\ 2 & 0 & 3 \end{pmatrix}$.

SECTION - IV

Answer any two questions from among the questions 32 to 35. These questions carry 15 marks each.

- 32. Use Green's function to solve $\frac{d^2y}{dx^2} + y = \csc x$ subject to the boundary conditions $y(0) = y\left(\frac{\pi}{2}\right) = 2$.
- 33. Given the vector field $\vec{a} = y\vec{i} x\vec{j} + z\vec{k}$, verify Stoke's theorem for the hemispherical surface $x^2 + y^2 + z^2 = a^2$, $z \ge 0$

34. Let
$$f(x) = \begin{cases} 1 & 0 < x < \frac{1}{2} \\ 0 & \frac{1}{2} < x < 1 \end{cases}$$

Find:

- (a) a Fourier sine series
- (b) a Fourier cosine series
- (c) a Fourier exponential series, whose period is 1
- 35. (a) Solve x-z=5, -2x+3y=1, x-3y+2z=-10 by the method of finding the inverse of the coefficient matrix.
 - (b) Find out whether the given vectors are dependent or independent; if they are dependent find a linearly independent subset.

$$(1, -2, 3), (1, 1, 1), (-2, 1, -4) (3, 0, 5)$$