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N – 2547

Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, March 2022

First Degree Programme under CBCSS

Mathematics

Complementary Course for Physics

MM 1331.1 – MATHEMATICS III – CALCULUS AND LINEAR ALGEBRA

(2018 Admission)

Time : 3 Hours   Max. Marks : 80

SECTION – I

All the **ten** questions are compulsory. They carry **1** mark each.

1. Define degree of an ODE.

2. What is the auxillary equation of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x$.

3. Write the general form of Bernoulli's equation.

4. Prove that $\vec{a} = (xy^2 + z)\vec{i} + (x^2y + 2)\vec{j}$ is conservative.

5. State Stoke's theorem.

6. Find the average value of $\sin x$ on $(-\pi, \pi)$.

7. What are the Fourier coefficients of an even function $f(x)$ in the interval $(-l, l)$.

P.T.O.

8. Evaluate $\begin{vmatrix} 1 & -5 & 2 \\ 7 & 3 & 4 \\ 2 & 1 & 5 \end{vmatrix}$.

9. Find the rank of the matrix $\begin{pmatrix} 1 & 1 & 2 \\ 2 & 4 & 6 \\ 3 & 2 & 5 \end{pmatrix}$.

10. Find the trace of the matrix $\begin{pmatrix} 1 & 0 & -1 \\ 2 & -3 & 0 \\ 1 & -3 & 2 \end{pmatrix}$.

SECTION – II

Answer **any eight** questions from among the questions **11** to **22**. These questions carry **2** marks each.

11. Solve $x\frac{dy}{dx}+3x+y=0$.

12. Solve $y=px+p^2$

13. Find the complementary function of the equation $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=4e^{-x}$.

14. Evaluate the line integral $I=\int \vec{a}\cdot d\vec{r}$ where $\vec{a}=(x+y)\vec{i}+(y-x)\vec{j}$ along the parabola $y^2=x$ from (1, 1) to (4, 2) in the xy-plane.

15. Find the vector area of the surface of the hemisphere $x^2+y^2+z^2=a^2$, $z\geq 0$, by evaluating the line integral $S=\frac{1}{2}\oint_C \vec{r}\times d\vec{r}$ around its perimeter.

16. Define even and odd functions and give examples.

17. What are the Dirichlet conditions for the existence of the Fourier series of a periodic function?

18. Define Fourier transform.

19. Find the direction of the line of intersection of the plane $x-2y+3z=4$ and $2x+y-z=5$.

20. Show that the functions $1, x, \sin x$ are linearly independent.

21. Find the inverse of the matrix $\begin{pmatrix} 2 & 1 \\ 0 & -3 \end{pmatrix}$.

22. Define symmetric matrix and prove that AA^T is a symmetric matrix for any matrix A.

SECTION – III

Answer **any six** questions from among the following questions **23** to **31**. These questions carry **4** marks each.

23. Solve : $\frac{dy}{dx} = -\frac{2}{y} - \frac{3y}{2x}$.

24. Solve : $\frac{dy}{dx} = (x+y+1)^2$.

25. Solve $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - 4y = 0$.

26. The vector field $\vec{F}$ is given by $\vec{F} = 2xz\vec{i} + 2yz^2\vec{j} + (x^2+2y^2z-1)\vec{k}$. Calculate $\nabla \times \vec{F}$ and deduce that $\vec{F}$ can be written $\vec{F} = \nabla\phi$. Determine the form of ϕ.

27. Show that the area of a region R enclosed by a closed curve C is given by $A = \frac{1}{2}\oint x\,dy - y\,dx = \oint_C x\,dy = -\oint_C y\,dx$.

28. Find the Fourier series of the function

$$f(x) = \begin{cases} 0 & -\pi < x < 0 \\ 1 & 0 < x < \pi \end{cases}$$

29. Expand $f(x) = \begin{cases} -1 & -l < x < 0 \\ 1 & 0 < x < l \end{cases}$ as a Fourier series.

30. Use Crammer's rule to solve $2x - z = 2$, $6x + 5y + 3z = 7$; $2x - y = 4$.

31. Find the eigen values and eigen vector of $\begin{pmatrix} 1 & 2 & 2 \\ 2 & 3 & 0 \\ 2 & 0 & 3 \end{pmatrix}$.

SECTION – IV

Answer **any two** questions from among the questions **32** to **35**. These questions carry **15** marks each.

32. Use Green's function to solve $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$ subject to the boundary conditions $y(0) = y\left(\frac{\pi}{2}\right) = 2$.

33. Given the vector field $\vec{a} = y\vec{i} - x\vec{j} + z\vec{k}$, verify Stoke's theorem for the hemispherical surface $x^2 + y^2 + z^2 = a^2$, $z \geq 0$

34. Let $f(x) = \begin{cases} 1 & 0 < x < \frac{1}{2} \\ 0 & \frac{1}{2} < x < 1 \end{cases}$

Find :

(a) a Fourier sine series

(b) a Fourier cosine series

(c) a Fourier exponential series, whose period is 1

35. (a) Solve $x - z = 5$, $-2x + 3y = 1$, $x - 3y + 2z = -10$ by the method of finding the inverse of the coefficient matrix.

(b) Find out whether the given vectors are dependent or independent; if they are dependent find a linearly independent subset.

$(1, -2, 3), (1, 1, 1), (-2, 1, -4)\ (3, 0, 5)$
