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N – 2556

Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, March 2022

First Degree Programme under CBCSS

Mathematics

Complementary Course for Physics

MM 1331.1 : MATHEMATICS III – CALCULUS AND LINEAR ALGEBRA

(2019 and 2020 Admission)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the **ten** questions are compulsory. They carry **1** mark each.

1. Find the degree of the ODE $\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^{3/2}+x^2y=0$

2. What is the auxillary equation of $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=e^x$.

3. Write the general form of Euler's linear equation.

4. Prove that $\vec{a}=(xy^2+z)\vec{i}+(x^2y+2)\vec{j}+x\vec{k}$ is conservative.

5. State Green's theorem.

6. Find the average value of $\sin^2 nx$ on $(-\pi, \pi)$.

P.T.O.

7. What are the Fourier coefficients of an even function $f(x)$ in the interval $(-l, l)$

8. Evaluate $\begin{vmatrix} 1 & -5 & 2 \\ 7 & 3 & 4 \\ 2 & 1 & 5 \end{vmatrix}$.

9. Find the rank of the matrix $\begin{pmatrix} 1 & 1 & 2 \\ 2 & 4 & 6 \\ 3 & 2 & 5 \end{pmatrix}$.

10. Find the trace of the matrix $\begin{pmatrix} 1 & 0 & -1 \\ -2 & 3 & 0 \\ 1 & -3 & 2 \end{pmatrix}$

SECTION – II

Answer **any eight** questions from among the questions **11** to **26**. These questions carry **2** marks each.

11. Solve $\dfrac{dy}{dx} = x + xy$.

12. Solve $x\dfrac{dy}{dx} + 3x + y = 0$.

13. Solve $y = px + p^2$.

14. Find the complementary function of the equation $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + y = 4e^{-x}$.

15. Find a solution of $(x^2 + x)\dfrac{dy}{dx}\dfrac{d^2y}{dx^2} - x^2y\dfrac{dy}{dx} - x\left(\dfrac{dy}{dx}\right)^2 = 0$.

16. Evaluate the line integral $I = \int a \cdot dr$ where $\vec{a} = (x + y)\vec{i} + (y - x)\vec{j}$ along the parabola $y^2 = x$ from $(1, 1)$ to $(4, 2)$ in the xy plane.

17. Find the vector area of the surface of the hemisphere $x^2 + y^2 + z^2 = a^2$, $z \geq 0$, by evaluating the line integral $S = \frac{1}{2}\oint_C \vec{r} \times d\vec{r}$ around its perimeter.

18. Find an expression for the angular momentum of a solid body rotating with angular velocity w about an axis through the origin.

19. Define even and odd functions and give examples.

20. What are the Dirichlet conditions for the existence of the Fourier series of a periodic function?

21. Define Fourier transforms.

22. Whether given set of equations has exactly one solution, no solution or an infinite, set of solutions $x - 2y + 13 = 0$, $y - 2x = 17$.

23. Find the direction of the line of intersection of the plane $x - 2y + 3z = 4$ and $2x + y - z = 5$.

24. Show that the functions $1, x, \sin x$ are linearly independent.

25. Find the inverse of the matrix $\begin{pmatrix} 2 & 1 \\ 0 & -3 \end{pmatrix}$.

26. Define symmetric matrix and prove that AA^T is a symmetric matrix for any matrix A.

SECTION – III

Answer **any six** questions from among the following questions **27** to **38**. These questions carry **4** marks each.

27. Solve : $\frac{dy}{dx} = -\frac{2}{y} - \frac{3y}{2x}$.

28. Solve : $\frac{dy}{dx} = (x + y + 1)^2$.

29. Solve $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-4y=0$.

30. Solve $(1-x^2)\frac{d^2y}{dx^2}-3x\frac{dy}{dx}-y=1$.

31. The vector field $\vec{F}$ is given by $\vec{F}=2xz\vec{i}+2yz^2\vec{j}+(x^2+2y^2z-1)\vec{k}$. Calculate $\nabla\times\vec{F}$ and deduce that $\vec{F}$ can be written $\vec{F}=\nabla\phi$. Determine the form of ϕ.

32. Show that the area of a region R enclosed by a closed curve C is given by $A=\frac{1}{2}\oint x\,dy-y\,dx=\oint_C x\,dy=-\oint_C y\,dx$.

33. Find the Fourier series of the function

$$f(x)=\begin{cases}0 & -\pi<x<0\\ 1 & 0<x<\pi\end{cases}$$

34. Expand $f(x)=\begin{cases}-1 & -l<x<0\\ 1 & 0<x<l\end{cases}$ as a Fourier series.

35. Find the equation of the plane through the three points $A(-1,1,1)$, $B(2,3,1)$, $C(0,1,-2)$.

36. Use Crammer's rule to solve $2x-z=2$; $6x+5y+3z=7$; $2x-y=4$.

37. Find the eigen values and any one eigen vector of $\begin{pmatrix}1 & 2 & 2\\ 2 & 3 & 0\\ 2 & 0 & 3\end{pmatrix}$.

38. Solve the set of homogeneous equations by row reducing the matrix $2x+3z=0$, $4x+2y+5z=0$, $x-y+2z=0$.

SECTION – IV

Answer **any two** questions from among the questions **39** to **44**. These questions carry **15** marks each.

39. Use Green's function to solve $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$ subject to the boundary conditions $y(0) = y\left(\frac{\pi}{2}\right) = 0$.

40. Express the equation $\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + (4x^2+6)y = e^{-x^2}\sin 2x$ in canonical form and hence find its general solution.

41. Given the vector field $\vec{a} = y\vec{i} - x\vec{j} + z\vec{k}$, verify Stoke's theorem for the hemispherical surface $x^2 + y^2 + z^2 = a^2$, $z \geq 0$

42. Let $f(x) = \begin{cases} 1 & 0 < x < \frac{1}{2} \\ 0 & \frac{1}{2} < x < 1 \end{cases}$

Find :

(a) a Fourier sine series

(b) a Fourier cosine series

(c) a Fourier series (exponential series, whose period) is 1

43. (a) Find the distance from the point $P(1, -2, 3)$ to the plane $3x - 2y + z + 1 = 0$.

(b) Find the distance between the lines $\vec{r} = \vec{i} - 2\vec{j} + (i - k)t$ and $\vec{r} = 2\vec{j} - \vec{k} + (\vec{j} - \vec{i})t$.

(c) Prove that $\begin{vmatrix} 1 & a & bc \\ 1 & b & ac \\ 1 & c & ab \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (c-a)(b-a)(c-b)$.