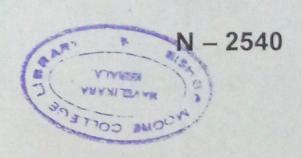
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Third Semester B.Sc. Degree Examination, March 2022 First Degree Programme under CBCSS

Mathematics

MM 1341 — ELEMENTARY NUMBER THEORY AND CALCULUS – I (2018 Admission)

Time: 3 Hours

Max. Marks: 80

SECTION - A

Answer all questions. Each question carries 1 mark.

- 1. Find five consecutive integers that are composites.
- State the prime number theorem.
- 3. If p is a prime and if $p \mid ab$, then prove that $p \mid a$ or $p \mid b$.
- 4. Express 3ABC_{sixteen} in base ten.
- 5. If $r(t) = t^2 i + e^t j (2\cos \pi t)k$, compute $\lim_{t \to 0} r(t)$.
- 6. Define the escape speed.
- 7. Determine whether the vector valued function $r(t) = t^2i + t^3j$ is smooth.
- 8. State the extreme value theorem.
- 9. Compute $\frac{dy}{dx}$ given that $x^3 + y^2x 3 = 0$.
- 10. Define the total differential of w = f(x, y, z) at (x_0, y_0, z_0) .

 $(10 \times 1 = 10 \text{ Marks})$

SECTION - B

Answer any eight questions. Each question carries 2 marks.

- 11. Find the primes such that their digits in the decimal values alternate between 0_{\$23} and 1s, beginning with and ending in 1.
- 12. Verify whether the LDEs 12x + 18y = 30 and 6x + 8y = 25 are solvable.
- 13. If $a \mid c$ and $b \mid c$, can we say that $ab \mid c$. Justify your answer.
- 14. Find the number of positive integers ≤3000 and divisible by 3, 5, or 7.
- 15. Estimate $\int_{0}^{1} r(t)dt$, where $r(t) = 2ti + t^{2}j (\sin \pi t)k$.
- 16. Write the formulas for acceleration and speed in 3 space.
- 17. Find T(s) by parameterizing the circle $r = a\cos i + a\sin t j$, $0 \le t \le 2\pi$, of radius a with counter clockwise orientation and centered at the origin.
- 18. Find the arc length of that portion of the circular helix $x = \cos t$, $y = \sin t$, z = t from t = 0 to $t = \pi$.
- 19. Determine maximum value of a directional derivative of $f(x,y) = x^2 e^y$ at (-2, 0) and the unit vector in the direction in which the maximum value occurs.
- 20. Consider the sphere $x^2 + y^2 + z^2 = 1$. Evaluate $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at $\left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)$.
- 21. Compute $\lim_{(x,y)\to(0,0)} (x^2 + y^2) \ln (x^2 + y^2)$.
- 22. Verify: If $F(x,y,z) = 2z^3 3(x^2 + y^2)z$, then $F_{xx} + F_{yy} + F_{zz} = 0$.

 $(8 \times 2 = 16 \text{ Marks})$

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SECTION - C

- Answer any six questions. Each question carries 4 marks.
- 23. Show that there are infinitely many primes of the form 4n + 3.
- 24. Find the number of trailing zeros in 234!.
- 25. Let a and b be positive integers. Derive a relationship between (a, b) and [a, b]. Also verify it for the integers 18 and 24.
- Let $r_1(t) = (\tan^{-1}t)i + (\sin t)j + t^2k$ and $r_2(t) = (t^2 t)i + (2t 2)j + (\ln t)k$. The graphs of $r_1(t)$ and $r_2(t)$ intersect at the origin. Find the degree measure of the acute angle between the tangent lines to the graphs of $r_1(t)$ and $r_2(t)$ at the origin.
- 27. Find r(t) given that $r'(t) = \langle 3, 2t \rangle$ and $r(1) = \langle 2, 5 \rangle$.
- 28. Compute the second order partial derivatives of $f(x,y) = x^2y^3 + x^4y$.
- 29. For the function $f(x,y) = -\frac{xy}{x^2 + y^{2'}}$ estimate the limit of f(x,y) as $(x,y) \to (0,0)$ along
 - (a) x axis
 - (b) y axis
 - (c) the line y = x
 - (d) the parabola $y = x^2$.
- 30. Given that $z = e^{xy}$, x = 2u + v, $y = \frac{u}{v}$, compute $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$.
- Derive the parametric equations of the tangent line to the curve of intersection of the paraboloid $z = x^2 + y^2$ and the ellipsoid $3x^2 + 2y^2 + z^2 = 9$ at the point (1, 1, 2).

SECTION - D

Answer any two questions. Each question carries 15 marks.

- 32. (a) A six digit positive integer is cut up in the middle into two three digit numbers. If the square of their sum yields the original, find the number.
 - (b) Solve the LDE 1076x + 2076y = 3076 by Euler's method.
- 33. (a) A geosynchronous orbit for a satellite is a circular orbit about the equator of the Earth in which the satellite stays fixed over a point on the equator. Use the fact that the Earth makes one revolution about its axis every 24 hours to find the altitude in miles of a communications satellite in geosynchronous orbit. Assume the earth to be a sphere of radius 4000 miles.
 - (b) In a projectile motion, derive the position function of the object in terms of its initial position and velocity.
- 34. Find the absolute maximum and minimum values of f(x,y) = 3xy 6x 3y + 7 on the closed triangular region R with vertices (0,0), (3,0) and (0,5).
- 35. Use Lagrange multipliers to determine the dimensions of a rectangular box, open at the top, having a volume of 32 ft³, and requiring the least amount of material for its construction.

 $(2 \times 15 = 30 \text{ Marks})$