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U – 2379

Reg. No. : .....

Name : .....

Fifth Semester B.Sc. Degree Examination, December 2024

First Degree Programme under CBCSS

Mathematics

Core Course

MM 1544 : DIFFERENTIAL EQUATIONS

(2018 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – I

Answer **all** the questions.

1. Find the degree of the differential equation : $\frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^2 + 6y + 10 = 0$.

2. Solve $y' = \frac{y}{x}$.

3. Define exact equations.

4. Find an integrating factor of the differential equation : $x\frac{dy}{dx} + 2y = 3$.

5. Find the Wronskian of $y'' + 4y = 0$.

6. Show that $y = 1 + \sin x$ is a solution of $y'' + y = 1$.

P.T.O.

7. Define standard form of Bernoulli's equation.

8. Solve $y'' + 2y' + 3y = 0$.

9. Form the differential equation of family of circles with center at origin and radius a.

10. Define singular solution of a differential equation.

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions.

11. Show the linear independence of x^2, $x^2 \ln x$ by using the Wronskian.

12. Find a particular solution of $y'' + 3y' + 2y = 5x^2$.

13. Write the auxiliary equation of the Euler-Cauchy equation $x^2y'' - 5xy' + 9y = 0$.

14. Form the differential equation whose general solution is $y = cx + c - c^3$.

15. Solve $\frac{dy}{dx} - y = e^x y^2$.

16. If exact, then solve the differential equation $(2x - 1)dx + (3y + 7)dy = 0$.

17. Find the orthogonal trajectories of $y = mx$.

18. Solve the initial value problem $y'' + y' - 2y = 0$, $y(0) = 4$, $y'(0) = -5$.

19. Find an integrating factor of $xy\,dx + (2x^2 + 3x^2 - 20)dy = 0$.

20. Solve $\frac{dy}{dx} + y \tan x = \cos^3 x$.

21. Determine the constant A such that the equation

$(Ax^2y + 2y^2)\,dx + (x^3 + 4xy)\,dy = 0$ is exact.

22. Solve $y\cos x dx + 3\sin x\,dy = 0$.

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions.

23. Given that $y = x$ is a solution of $(x^2+1)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$, find a linearly independent solution by reducing the order.

24. Solve $(y^2 + yx)dx + x^2dy = 0$.

25. Solve the differential equation $y'' - 10y' + 25y = 30x + 3$ by undertermined coefficients.

26. Solve $x^2y'' + xy' + y = 0$ subject to $y(1) = 1,\ y'(1) = 2$.

27. Make the following equation exact and hence solve

$(xy^3 + y)dx + (x^2y^2 + x + y^4)dy = 0$

28. Solve $(D^2 + 4)y = 3\sin 2x$.

29. Solve $(x^2 - 4xy - 2y^2)dx + (y^2 - 4xy - 2x^2)dy = 0$.

30. Solve $x\frac{dy}{dx} + (3x+1)y = e^{-3x}$.

31. Show that for a second order homogeneous linear differential equation, any linear combination of 2 solutions on an open interval I is again a solution of the differential equation on I.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions.

32. Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 2e^x - 10\sin x.$$

33. Solve $\frac{d^2y}{dx^2} + y = \tan x$ using the method of variation of parameters.

34. (a) Find the orthogonal trajectories of confocal parabolas $y^2 = 4(x+a)$.

(b) Solve $(D^2 - 10D + 25)y = 0$.

35. (a) Solve $\frac{dy}{dx} + \frac{x-2y}{2x-y} = 0$.

(b) Solve the Bemoulli's equation : $\frac{dy}{dx} + y = xy^3$.

(2 × 15 = 30 Marks)