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Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2024

First Degree Programme under CBCSS

Mathematics

Core Course

MM 1541 — REAL ANALYSIS — I

(2018 Admission Onwards)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the **first ten** questions are compulsory. They carry **1** mark each.

1. Let $f(x)=x^2$. If $A=[0,2]$, $B=[1,4]$, find $f(A)$ and $f(B)$.

2. Show that $|ab| = |a|\,|b|$ for all $a, b \in R$.

3. Give an example for a one-one function from $(-1,1)$ onto R.

4. Find infimum of the set $\left\{2+\frac{3}{n};\ n\in N\right\}$.

5. Write the first five terms of the sequence defined inductively by $x_1=2, x_{n+1}=\frac{x_n+1}{2}$.

P.T.O.

6. Find $\lim\left(\frac{2}{5}\right)^n$.

7. Give an example for a monotone sequence that is not Cauchy.

8. Check whether $(1,5)$ is compact.

9. State true or false: Union of two connected sets is connected. Justify your answer.

10. Define nowhere dense set and give example.

(10 × 1 = 10 Marks)

SECTION – II

Answer **any eight** questions. These questions carry 2 marks each.

11. Show that the set $E=\{2n; n\in N\}$ is countable.

12. Show that $\sqrt[3]{2}$ is algebraic.

13. Show that the sequence $\left(1,\frac{1}{2},3,\frac{1}{4},\ldots\right)$ is divergent.

14. Give an example of a series which is convergent but not absolutely convergent.

15. Show that the sequence $\left(\frac{1}{n}\right)$ is Cauchy.

16. Show that every convergent sequence is bounded.

17. Show that $\lim(a_n+b_n)=\lim a_n+\lim b_n$.

18. Give an example for an unbounded sequence which contain a subsequence that is Cauchy.

19. If $\sum_{k=1}^{\infty} a_k = A$, show that $\sum_{k=1}^{\infty} ca_k = cA$.

20. Show that $A = \left\{\frac{1}{n};\ n \in N\right\}$ is not closed.

21. For any $A \subseteq R$, show that the closure A is the smallest closed set containing A.

22. Give an example of a disconnected set whose closure is connected.

(8 × 2 = 16 Marks)

SECTION – III

Answer **any six** questions. These questions carry **4** marks each.

23. State and prove Nested Interval property.

24. Given any number $x \in R$, show that there exist $n \in N$ satisfying $n > x$.

25. Test for convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n}$.

26. Construct a sequence that converges to $\sqrt{2}$.

27. Show that two real numbers a, b are equal if and only if for every real number $\varepsilon > 0$, $|a-b| < \varepsilon$.

28. Define geometric series. Discuss its convergence.

29. Show that if a set $K \subseteq R$ is compact. then it is closed and bounded.

30. Show that a point x is a limit point of a set A if and only if $x = \lim a_n$ for some sequence (a_n) contained in A satisfying $a_n \neq x$ for all $n \in N$.

31. Construct an open cover for (0,1) in such a way that it has no finite sub cover.

(6 × 4 = 24 Marks)

SECTION – IV

Answer **any two** questions. These questions carry **15** marks each.

32. (a) Show that the set Q is countable.

(b) State and prove Canto's theorem.

33. (a) State and prove Cauchy condensation test.

(b) Let $Y=(y_n)$ be defined inductively by $y_1=1, y_{n+1}=\frac{1}{4}(2y_n+3)$ for $n\geq 1$. Find $\lim Y$.

34. (a) Show that a sequence converges if and only if it is a Cauchy sequence.

(b) Discuss the convergence of the series $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{2^n}$.

35. Show that a non empty perfect set is uncountable.

(2 × 15 = 30 Marks)