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U – 2377

Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2024

First Degree Programme under CBCSS

Mathematics

Core Course

MM 1542 : COMPLEX ANALYSIS – I

(2018 Admission Onwards)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. They carry **1** mark each.

1. Find i^{62}.
2. Describe the set of points Rez $\geq$ 4.
3. Find Arg10i.
4. State DeMoivre's formula.
5. Find boundary of $0 < |z-2| < 3$.
6. Define analytic function in a domain.
7. State Morera's theorem.
8. Define sin z.
9. Define complex exponent z^{α}, where $z \neq 0$ and α is a complex constant.
10. Evaluate $\int_0^1 (2t + it^2)\,dt$.

(10 × 1 = 10 Marks)

P.T.O.

SECTION – II

Answer any **eight** questions. These questions carry **2** marks each.

11. Write the number $\left((3-i)^2-3\right)i$ in the form $a+bi$.

12. Write $\left(\sqrt{3}-i\right)^2$ in polar form.

13. Find $(1+i)^{24}$.

14. Show that $\left|e^z\right| \leq 1$, if $Rcz \leq 0$.

15. Find points at which $f(z)=\dfrac{iz^3+2z}{z^2+1}$ is not analytic.

16. Prove that e^{iz} is periodic with a period 2π.

17. Find all poles and their multiplicities of the function $f(z)=\dfrac{z^2+1}{(z-2)(z-3)^4}$.

18. Describe analyticity of $\log z$.

19. Find the Taylor form of the polynomial $g(z)=(z-1)(z-2)^3$ centred at $z=2$.

20. Define simply connected domain. Give an example.

21. Find $\int_C \dfrac{\cos z}{z^2+z-12}dz$, where C is $|z|=2$.

22. Compute $\int_C \cos z dz$, where C is the contour formed by upper semi circle of $|z|=1$ from -1 to 1 followed by line segment from 1 to $2+i$. (Use independence of path).

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions. These questions carry **4** marks each.

23. Find all values of $(-16)^{\frac{1}{4}}$.

24. Write $f(z)=\dfrac{2z^2+3}{|z-1|}$ in $u(x, y)+iv(x, y)$ form.

25. Prove that $f(z)=e^z$ is entire and find its derivative.

26. Show that if f is analytic in a domain D and either $\operatorname{Re}(f(z))$ or $\operatorname{Im}(f(z))$ is constant, then $f(z)$ must be constant.

27. A polynomial $p(z)$ of degree 4 has zeros at the points -1, $3i$ and $-3i$ of respective multiplicities 2, 1 and 1. If $p(1)=80$, find $p(z)$.

28. Find all values of $(-2)^i$.

29. Show that if $z_1=i$ and $z_2=i-1$, then $\log(z_1z_2)\neq\log z_1+\log z_2$.

30. If γ is the vertical line segment from $z=R(R>0)$ to $z=R+2\pi i$, then show that

$$\left|\int_\gamma \frac{e^{3z}}{1+e^z}dz\right| \leq \frac{2\pi e^{3R}}{e^R-1}.$$

31. Compute $\int_\Gamma \bar{z}dz$, where (a) Γ is in the circle $|z|=2$ traversed once counterclockwise (b) Γ is the circle $|z|=2$ traversed once clockwise.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions. These questions carry **15** marks each.

32. (a) Evaluate $\int_0^{2\pi}\sin^4\theta\, d\theta$.

(b) State and prove the necessary conditions (Cauchy-Riemann equations) for a function to be analytic at a point.

33. Let C be the perimeter of the square with vertices at the points $z=0$, $z=1$, $z=1+i$ and $z=i$ traversed once in that order. Show that $\int_C e^z dz=0$.

34. (a) State Cauchy's integral formula.

(b) Let C be the circle $|z| = 2$ traversed once in the positive sense. Compute each of the following integrals.

(i) $\int_C \frac{\cos z}{z^3 + 9z} dz$

(ii) $\int_C \frac{\sin z}{z^2 (z - 4)} dz$

(iii) $\int_C \frac{5z^2 + 2z + 1}{(z - i)^3} dz$.

35. State and prove fundamental theorem of Algebra.

(2 × 15 = 30 Marks)