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N – 2532

Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, March 2022.

First Degree Programme under CBCSS

Statistics

Complementary Course for Mathematics

ST 1331.1 – PROBABILITY DISTRIBUTIONS AND THEORY OF ESTIMATION

(2014 – 2017 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. . Each carries **1** mark.

1. If $X \sim N(0,1)$, then the distribution of the square of X is ——————

2. X is the number shown when a fair die is tossed, X follows —————— distribution.

3. A sample of size n was taken from a population following $N(\mu, \sigma^2)$. The standard error of the sample mean $\overline{X}$ is ——————

4. The moment generating function of a random variable that follows χ^2 distribution with 2 degrees of freedom is ——————

5. If t_n is a consistent estimator of θ, then as $n \to \infty, V(t_n)$ tends to ——————

P.T.O.

6. For a normal curve, the quartile deviation (QD), mean deviation (MD) and standard deviation (SD) are in the ratio ________

7. What are the points of inflexion of the normal curve?

8. A single value of an estimator for a population parameter θ is called its ________ estimate.

9. What do you mean by sampling distribution of a statistic?

10. Let $X_1, X_2, \ldots X_n$ be a random sample from $U[0,\theta]$. The mle of θ is ________

(10 × 1 = 10 Marks)

SECTION – B

Answer **any eight** questions. Each carries **2** marks.

11. Show that for the geometric distribution, $p(x+1) = qp(x)$.

12. If $X \sim N(\mu_1, \sigma_1)$ and $Y \sim N(\mu_2, \sigma_2)$, obtain the distribution of $Y = aX + bY$, where X and Y are independent.

13. X is a random variable with a continuous distribution. Find the distribution of $Y = F(X)$ where $F(x)$ is the distribution function of X.

14. Define convergence in probability.

15. In a town 10 accidents took place in a span of 100 days. Find the probability of 3 or more accidents in a day if it follows Poisson distribution.

16. State Bernoulli law of large numbers.

17. Derive the mean of a binomial distribution.

18. Differentiate between parameter and statistic.

19. Give any two applications of Chi square distribution.

20. Show that sample mean is an unbiased estimate of the population mean when samples are taken from a normal population.

21. State central limit theorem.

22. State Neyman's condition for sufficiency.

(8 × 2 = 16 Marks)

SECTION – C

Answer **any six** questions, Each carries **4** marks.

23. Obtain the moment generating function of Poisson distribution with parameter λ.

24. Prove the recurrence relation between the moments of binomial distribution

$$\mu_{r+1} = pq[nr\mu_{r-1} + \frac{d\mu_r}{dp}],$$

Where $\mu_{r-1}, \mu_r, \mu_{r+1}$ are the central moments about the parameters n and p.

25. If $x_1, x_2, \ldots x_n$ are independent random variables having exponential distribution with parameter λ, find the distribution of $Y = \sum_{i=1}^{n} X_i$.

26. Explain the method of maximum likelihood estimation (MLE) of parameters. Write any two properties of MLE.

27. Find the sufficient statistics for Gamma distribution with parameter α and β.

28. Show that $t = \dfrac{n\overline{x}}{n+1}$ is a consistent estimator of λ, where $\overline{x}$ is the mean of samples of size n taken from a Poisson population.

29. Find a lower limit for the variance of any unbiased estimator of θ, where $f(x:\theta) = \theta e^{-\theta x}$, $x > 0$.

30. If $E(X)=3, E(X^2)=13$, use Chebychev's inequality to find a lower bound for $P\{-2 < X < 8\}$.

31. Explain the relation between normal, chi square, t and F distributions.

(6 × 4 = 24 Marks)

SECTION – D

Answer **any two** questions. Each carries **15** marks.

32. (a) Define beta distribution of first kind and second kind. Obtain their means.

(b) If X follows beta distribution of first kind with parameters p and q, show that $Y = \frac{X}{1-X}$ follows beta distribution of second kind.

33. (a) State and prove Weak law of large numbers.

(b) For a sequence of random variable $\{X_k\}$, given that $P(X_k = -2^k) = 2^{-(2k+1)} = P(X_k = 2^k), P(X_k = 0) = 1 - 2^{-(2k+1)}$. Examine if the law of large numbers hold for this Sequence.

34. Derive 95 % confidence interval for mean from normal distribution when (1) σ is known (2) σ is unknown.

35. Let $x_1, x_2, \ldots x_n$ be a random sample from $N(\mu, \sigma^2)$ population. Find sufficient estimators for

(a) μ when σ^2 is known

(b) σ^2 when μ is known

(c) μ and σ^2 when both are unknown.

(2 × 15 = 30 Marks)