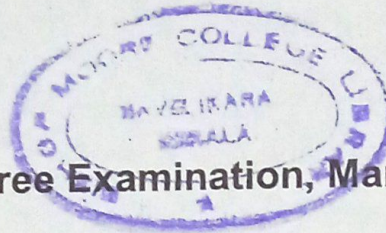


Reg. No. : .....

Name : .....



Third Semester B.Sc. Degree Examination, March 2022

First Degree Programme under CBCSS

Mathematics

MM 1341 : ELEMENTARY NUMBER THEORY AND CALCULUS — I

(2019 & 2020 Admission)

Time : 3 Hours

Max. Marks : 80

PART – A

Answer *all* questions. Each question carries 1 mark.

1. Find five consecutive integers that are composites.
2. State the prime number theorem.
3. If  $p$  is a prime and if  $p \mid ab$ , then prove that  $p \mid a$  or  $p \mid b$ .
4. Express  $3ABC_{\text{sixteen}}$  in base ten.
5. If  $r(t) = t^2i + e^tj - (2\cos \pi t)k$ , compute  $\lim_{t \rightarrow 0} r(t)$ .
6. Define the escape speed.
7. Determine whether the vector-valued function  $r(t) = t^2i + t^3j$  is smooth.

8. State the extreme-value theorem.
9. Compute  $\frac{dy}{dx}$  given that  $x^3 + y^2x - 3 = 0$ .
10. Define the total differential of  $w = f(x, y, z)$  at  $(x_0, y_0, z_0)$ .

(10 × 1 = 10 Marks)

PART – B

Answer **any eight** questions. Each question carries **2** marks.

11. Find the primes such that their digits in the decimal values alternate between 0s and 1s, beginning with the ending in 1.
12. Show that every integer  $n \geq 2$  has a prime factor.
13. Verify whether the LDEs  $12x + 18y = 30$  and  $6x + 8y = 25$  are solvable.
14. If  $a | c$  and  $b | c$ , can we say that  $ab | c$ ? Justify your answer.
15. Find the number of positive integers  $\leq 3000$  and divisible by 3, 5, or 7.
16. Show that 111 cannot be a square in any base.
17. State any two rules of differentiation of vector-valued functions.
18. Estimate :  $\int_0^1 r(t) dt$ , where  $r(t) = 2ti + t^2j - (\sin \pi t) k$ .
19. Write the formulas for acceleration and speed in 3-space.

20. Find  $T(s)$  by parameterizing the circle  $r = a \cos t i + a \sin t j$ ,  $0 \leq t \leq 2\pi$ , of radius  $a$  with counter clockwise orientation and centered at the origin.
21. Find the arc length of that portion of the circular helix  $x = \cos t$ ,  $y = \sin t$ ,  $z = t$  from  $t = 0$  to  $t = \pi$ .
22. Determine maximum value of a directional derivative of  $f(x, y) = x^2 e^y$  at  $(-2, 0)$  and the unit vector in the direction in which the maximum value occurs.
23. Consider the sphere  $x^2 + y^2 + z^2 = 1$ . Evaluate  $\frac{\partial z}{\partial x}$  and  $\frac{\partial z}{\partial y}$  at  $\left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)$ .
24. Compute  $\lim_{(x, y) \rightarrow (0, 0)} (x^2 + y^2) \ln(x^2 + y^2)$ .
25. Verify : If  $F(x, y, z) = 2z^3 - 3(x^2 + y^2)z$ , then  $F_{xx} + F_{yy} + F_{zz} = 0$ .
26. State the second partials test.

(8 × 2 = 16 Marks)

PART – C

Answer **any six** questions. Each question carries **4** marks.

27. Show that there are infinitely many primes of the form  $4n + 3$ .
28. Find the number of trailing zeros in  $234!$ .
29. Let  $b$  be an integer  $\geq 2$ . Suppose  $b + 1$  integers are randomly selected. Prove that the difference of two of them is divisible by  $b$ .
30. Let  $a$  and  $b$  be positive integers. Derive a relationship between  $(a, b)$  and  $[a, b]$ . Also verify it for the integers 18 and 24.

31. Let  $r_1(t) = (\tan^{-1} t)i + (\sin t)j + t^2k$  and  $r_2(t) = (t^2 - t)i + (2t - 2)j + (\ln t)k$ . The graphs of  $r_1(t)$  and  $r_2(t)$  intersect at the origin. Find the degree measure of the acute angle between the tangent lines to the graphs of  $r_1(t)$  and  $r_2(t)$  at the origin.
32. Find  $r(t)$  given that  $r'(t) = \langle 3, 2t \rangle$  and  $r(1) = \langle 2, 5 \rangle$ .
33. Show that the trajectory of a projectile is a parabolic path.
34. Derive Kepler's second law.
35. Compute the second-order partial derivatives of  $f(x, y) = x^2y^3 + x^4y$ .
36. For the function  $f(x, y) = \frac{xy}{x^2 + y^2}$ , estimate the limit of  $f(x, y)$  as  $(x, y) \rightarrow (0, 0)$  along.
- x-axis
  - y-axis
  - the line  $y = x$
  - the parabola  $y = x^2$ .
37. Given that  $z = e^{xy}$ ,  $x = 2u + v$ ,  $y = \frac{u}{v}$ , compute  $\frac{\partial z}{\partial u}$  and  $\frac{\partial z}{\partial v}$ .
38. Derive the parametric equations of the tangent line to the curve of intersection of the paraboloid  $z = x^2 + y^2$  and the ellipsoid  $3x^2 + 2y^2 + z^2 = 9$  at the point  $(1, 1, 2)$ .

(6 × 4 = 24 Marks)

PART – D

Answer **any two** questions. Each question carries **15** marks.

39. (a) A six-digit positive integer is cut up in the middle into two three-digit numbers. If the square of their sum yields the original, find the number.
- (b) Solve the LDE  $1076x + 2076y = 3076$  by Euler's method.
40. (a) State and prove the fundamental theorem of arithmetic.
- (b) Explain the Euclidean algorithm and evaluate  $(4076, 1024)$ .
41. (a) A geosynchronous orbit for a satellite is a circular orbit about the equator of the Earth in which the satellite stays fixed over a point on the equator. Use the fact that the Earth makes one revolution about its axis every 24 hours to find the altitude in miles of a communications satellite in geosynchronous orbit. Assume the earth to be a sphere of radius 4000 miles.
- (b) In a projectile motion, derive the position function of the object in terms of its initial position and velocity.
42. (a) A particle moves through 3-space in such a way that its velocity is  $v(t) = i + tj + t^2k$ . Find the coordinates of the particle at time  $t = 1$  given that the particle is at the point  $(-1, 2, 4)$  at time  $t = 0$ .
- (b) Find  $k(t)$  for the circular helix  $x = acost, y = asint, z = ct$  where  $a > 0$ .

43. Find the absolute maximum and minimum values of  $f(x, y) = 3xy - 6x - 3y + 7$  on the closed triangular region  $R$  with vertices  $(0, 0)$ ,  $(3, 0)$  and  $(0, 5)$ .
44. Use Lagrange multipliers to determine the dimensions of a rectangular box, open at the top, having a volume of  $32 \text{ ft}^3$ , and requiring the least amount of material for its construction.

**(2 × 15 = 30 Marks)**

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