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Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, March 2022

First Degree Programme under CBCSS

Mathematics

Complementary Course for Chemistry and Polymer Chemistry

MM 1331.2 – MATHEMATICS III – VECTOR ANALYSIS AND THEORY OF EQUATIONS

(2014 & 2017 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the **first ten** questions are compulsory. They carry **1** mark each.

1. Express the parametric equations $x = 3\cos t$, $y = 2t + \cos 2t$ as a single vector equation of the form.

2. Let $r(t) = t^2 i + e^t j - (2\cos \pi t)k$. Then find $\lim_{t \to 0} r(t)$.

3. If $r(t)$ is a differentiable vector-valued function in 2-space or 3-space and $\|r(t)\|$ is constant for all t, then prove that $r(t) \cdot r'(t) = 0$.

4. Find the arc length parametrization of the line $x = 1 + t$, $y = 3 - 2t$, $z = 4 + 2t$ that has the same direction as the given line and has reference point (1,3,4).

5. Describe the level surface of $f(x, y, z) = x^2 + y^2 + z^2$.

6. If the function $\phi(x, y, z) = xy + yz + xz$ is a potential function for the vector field F. Then find F.

P.T.O.

7. Use Descartes rule of sign to show that $x^7 - 3x^4 + 2x^3 - 1 = 0$ has at least four imaginary roots.

8. State the fundamental Theorem of Algebra.

9. Solve the equation $x^3 + 3x^2 - 6x - 8 = 0$ given that the three roots are in geometric sequence.

10. Find quadratic equation with the roots $x = -2 + i\sqrt{5}$ and $x = -2 - i\sqrt{5}$.

(10 × 1 = 10 Marks)

SECTION – II

Answer **any eight** questions. These questions carry **2** marks each.

11. Find the curvature of a circle of radius a.

12. A particle moves through 3-space in such a way that its velocity is $v(t) = i + tj + t^2k$. Find the coordinates of the particle at time $t = 1$ given that the particle is at the point (–1, 2, 4) at time $t = 0$.

13. Find $r(t)$ given that $r'(t) = \langle 3, 2t \rangle$ and $r(1) = \langle 25 \rangle$.

14. If $r = xi + yj + zk$ then prove that $curl\, r = 0$.

15. Prove that the vector field $3y^4z^2i + 4x^3z^3j - 3x^2y^2k$ is solenoidal

16. Find the area of the surface extending upward from the circle $x^2 + y^2 = 1$ in the xy plane to the parabolic cylinder $z = 1 - x^2$.

17. Find the work done by the force field F on a parabola that moves along the curve $C F(x, y) = xyi + x^3j$ where $C : x = y^2$ from (0, 0) to (1, 1)

18. A particle moves along a circular path in such a way that its x- and y-coordinates at time t are $x = 2\cos t$, $y = 2\sin t$. Find the instantaneous velocity and speed of the particle at time t and at $t = \frac{\pi}{4}$.

19. Solve the equation $x^3 - 8x^2 + 9x + 18 = 0$ given that the sum of two the roots is 5.

20. State the Descartes's rule of signs. Determine the number of positive and negative zeros of equation $6x^4 + 5x^3 - 14x^2 + x + 2 = 0$.

21. Solve $2x^3 + x^2 - 7x - 6 = 0$, given that the difference between of the two roots is 3.

22. Use Descartes' Rule of Signs to determine possibilities for the zeros of the polynomial $f(x) = x^5 + 4x^4 - 2x^3 - 14x^2 - 3x - 18$.

($8 \times 2 = 16$ Marks)

SECTION – III

Answer **any six** questions. These questions carry **4** marks each.

23. Find parametric equations of the tangent line to the circular helix $c = \cos t$, $y = \sin t$, $z = t$ where $t = t_0$, and use that result to find parametric equations for the tangent line at the point where $t = \pi$.

24. Find the divergence and curl of the vector field $F(x, y, z) = x^2yi + 2y^3j + 3zk$.

25. Find the directional derivative of $f(x, y) = xe^y + \cos(xy)$ at the point (2, 0) in the direction of the vector $i + j$.

26. Evaluate the surface integral $\iint_\sigma xz \, dS$ where σ is the part of the plane $x + y + z = 1$ that lies in the first octant.

27. Find the flux of the vector field $F(x, y, z) = zk$ across the outward oriented sphere $x^2 + y^2 + z^2 = a^2$.

28. Suppose that a curved lamina σ with constant density $\delta(x, y, z) = \delta_0$ is the portion of the paraboloid $z = x^2 + y^2$ below the plane $z = 1$. Find the mass of the lamina.

29. Show that the divergence of the inverse-square field $F(x, y, z) = \frac{c}{(x^2+y^2+z^2)^{\frac{3}{2}}}(xi+yj+zk)$ is zero.

30. Use the Divergence Theorem to find the outward flux of the vector field $F(x, y, z) = x^3 i + y^3 j + z^3 k$ across the surface of the region that is enclosed by the circular cylinder $x^2 + y^2 = 9$ and the planes $z = 0$ and $z = 2$.

31. Describe the Newton-Raphson procedure of finding the solution of a general equation $f(x) = 0$.

($6 \times 4 = 24$ Marks)

SECTION – IV

Answer **any two** questions. These questions carry **15** marks each.

32. Verify the Green's theorem for the line integral $\oint(xy\,dx + x^2 dy)$ where C is the curve enclosing the region bounded by the parabola $y = x^2$ and the line $y = x$, in counter clockwise direction.

33. Let $F(x, y) = e^y i + xe^y j$ denote a force field in the xy-plane.

    (a) Verify that the force field F is conservative on the entire xy-plane.

    (b) Find the work done by the field on a particle that moves from (1, 0) to (–1, 0) along the semicircular path C.

34. The graph of the vector equation $r = 2\cos ti + 3\sin tj (0 \le t \le 2\pi)$ is an ellipse. Find the curvature of the ellipse at the endpoints of the major and minor axes.

35. Show that $f(x) = x^3 + 4x^2 - 10 = 0$ has a root in [1, 2] and use the bisection method to determine an approximation to the root that with 10^{-6}.

($2 \times 15 = 30$ Marks)
