(Pages : 4)

M – 4625

Reg. No. : ....

Name : ....

Fourth Semester B.Sc. Degree Examination, February 2022

First Degree Programme Under CBCSS

Mathematics

Complementary Course for Chemistry /Polymer Chemistry

MM 1431.2 — MATHEMATICS IV – DIFFERENTIAL EQUATIONS, VECTOR CALCULUS AND ABSTRACT ALGEBRA

(2019 Admission)

Special Examination

Time : 3 Hours　　　　Max. Marks : 80

I. All the first 10 questions are compulsory. Each carries 1 mark.

1. Solve $\frac{dy}{dx}+2y=0$

2. Write the standard form of linear first order differential equation.

3. Check whether $ydx + xdy$ is exact or not.

4. True / False: y_1 and y_2 are independent implies $W(y_1, y2) = 0$.

5. Find the gradient of $f(x,y) = x - 2y$.

6. State Green's theorem in a plane.

7. Define Conservative field.

8. True/False: Every cyclic group is Abelian.

P.T.O.

9. State Lagrange's theorem.

10. Define characters of a representation of a group.

(10 × 1 = 10 Marks)

II. Answer any **eight** questions from among the questions 11 to 26. Each carries **2** marks.

11. Find the order and degree of the differential equation $\frac{d^2y}{dx^3} + x\left(\frac{dy}{dx}\right)^{\frac{3}{2}} = 0$

12. Solve $\frac{dy}{dx} = x + xy$.

13. Find the general solution of $y = px + p^2$.

14. Find the complementary function of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 4e^x$.

15. Write the standard form of Legendre's linear equation and Euler linear equation.

16. Solve $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 4 = 0$.

17. Find div F, where for $F = 2xy\boldsymbol{i} + 2yx^2\boldsymbol{j}$.

18. Write the integral form for grad, div and curl.

19. Evaluate the line integral $I = \int_C 3x^2 - 2y + z$, where C is the line segment joining (0,0,0) and (2,2,2).

20. Find an expression for the angular momentum of a solid body rotating with angular velocity ω about an axis through the origin

21. Draw the group table for the set {1,-1,i,–i} under ordinary multiplication of complex numbers.

22. Define representation of a group G with an example.

23. Define a group.

24. Define an equivalence relation.

25. State Division Axiom in group theory.

26. Define the index of H in G.

(8 × 2 = 16 Marks)

III. Answer any **six** questions from among the questions 27 to 38. Each carries **4** marks.

27. Solve $x\frac{dy}{dx} + 3x + y = 0$

28. Solve $\frac{dy}{dx} + 2xy = 4x$.

29. Find the general solution of the equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x$.

30. Solve $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - 4y = 0$.

31. Calculate $\nabla \times F$ for $F = 2xz\,\mathbf{i} + 2yz^2\mathbf{j} + (x^2 + 2y^2z - 1)\mathbf{k}$.

32. Find the volume enclosed between a sphere of radius a centered in the origin and a circular cone of half angle α with its vertices at the origin.

33. Using Green's theorem, find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

34. Evaluate the line integral $I = \oint_C x\,dy$, where C is the circle in the xy-plane defined by $x^2 + y^2 = a^2$, $z = 0$.

35. Prove that the identity element in a group is unique.

36. Show that the integer (mod 10) under addition is a group.

37. Prove that two cosets are either disjoint or identical.

38. If n_μ is the dimension of the μ^{th} irrep of a group G, then prove that $\sum_\mu n_\mu{}^2 = g$, where g is the order of G.

(6 × 4 = 24 Marks)

IV. Answer any **two** questions from among the questions 39 to 44. **Each** carries **15** marks.

39. (a) Solve $\frac{dy}{dx} = \frac{-1}{2yx}\left(y^2 + \frac{2}{x}\right)$

(b) Solve $\frac{dy}{dx} + \frac{y}{x} = 2x^3y^4$

40. (a) Solve $\frac{d^2y}{dx^2} + y = \cos ecx$.

(b) Using the variation of parameters method to solve $\frac{d^2y}{dx^2} + y = \cos ecx$ subject to the boundary conditions $y(0) = y\left(\frac{\pi}{2}\right) = 0$.

41. Evaluate the line integral $I = \int_C \mathbf{a}.dr$, where $\mathbf{a} = (x + y)\,\mathbf{i} + (y - x)\,\mathbf{j}$, along each of the paths in the xy plane, namely

(a) the parabola $y^2 = x$ from (1,1) to (4,2)

(b) the curve$x = 2u^2 + u + 1$, $y = 1 + u^2$, from (1,1) to (4,2).

(c) the line$y = 1$ from (1,1) to (4,1) followed by the line$x = 4$ from (4,1) to (4,2).

42. Show that $Q = (3x^2(y + z) + y^3 + z^3)\,\mathbf{i} + (3y^2(z + x) + z^3 + x^3)\,\mathbf{j} + (3z^2(x + y) + x^3 + y^3)\,\mathbf{k}$ is a conservative field. Find its potential function. Evaluate $\int_{(1,-1,1)}^{(2,1,2)} Q.dr$.

43. Explain the elements of a group of two dimensional operations which transform an equilateral triangle into itself, by drawing a sequence of model triangles. Draw the group table.

44. Determine the irreps contained in the representation of the group 3 m in the vector space spanned by the function x^2, y^2, xy.

(2 × 15 = 30 Marks)
